

Bose-Einstein condensates in disordered potentials

Leonardo Fallani, Chiara Fort, and Massimo Inguscio

*LENS European Laboratory for Nonlinear Spectroscopy
and Dipartimento di Fisica, Università di Firenze
Via Nello Carrara 1, 50019 Sesto Fiorentino (FI), Italy*

Abstract

The interplay between disorder and interactions is a *leit-motiv* of condensed matter physics, since it constitutes the driving mechanism of the metal-insulator transition. Bose-Einstein condensates in optical potentials are proving to be powerful tools to quantum simulate disordered systems. We will review the main experimental and theoretical results achieved in the last few years in this rapidly developing field.

Key words:

Ultracold quantum gases, Disordered systems

PACS: 05.30.Jp, 03.75.Kk, 03.75.Lm, 42.25.Dd

Contents

1	Introduction	2
2	How to produce a disordered potential	4
2.1	Speckle patterns	4
2.2	Multichromatic lattices	7
2.3	Other methods	9
3	Weakly interacting regime	10
3.1	A Bose-Einstein condensate in a disordered potential	13
3.1.1	Static properties	13
3.1.2	Collective excitations	15
3.2	The quest for Anderson localization	16

3.2.1	Localization in a speckle potential	16
3.2.2	Localization in a bichromatic lattice	20
3.3	Further directions	24
3.4	Anderson localization: the state of the art	26
4	Strongly interacting regime	28
4.1	The quest for Bose glass	31
4.1.1	Excitation spectrum and coherence properties	31
4.1.2	Noise correlations	33
4.2	Experiments with atomic mixtures	34
5	Conclusions	36
6	Acknowledgments	37

1 Introduction

In Nature many processes occur in an ordered way. Indeed, ordered configurations are often the ones minimizing the total energy of the system. A prominent example of this tendency towards order is given by the growth of a crystal, where the atoms arrange themselves in a spatially periodic configuration building up an ordered lattice. The physics of transport of electrons in a metal heavily relies on the periodicity of this lattice. However, when crystalline solids are studied on a sufficiently small length scale, one realizes that impurities and defects are always present, which may affect in a substantial way the transport of the electrons. Disorder is indeed an intrinsic property of all the real systems. In the last 50 years the effects of disorder on transport phenomena have been extensively studied in the context of both statistical and condensed matter physics.

Despite the very general interest in understanding the physics of disorder in condensed matter systems, still many questions remain open and unsolved, even from the theoretical point of view. As a matter of fact, the theoretical description of periodic systems, as perfect crystals, is much easier than the one for disordered system as disordered lattices or glasses. The problems that arise are related to the fact that the effects of disorder cannot be theoretically treated in a perturbative way: even a small amount of disorder can produce dramatic changes in the physical properties of the system under investigation.

In 1958, P. W. Anderson published a seminal paper (Anderson, 1958) in which he showed under which conditions non-interacting electrons in a disordered metal can either move through the system, or be localized. It was soon realized that Anderson localization is a much more general phenomenon holding for the propagation of generic classical waves in disordered media. Localization is a coherent effect that arises from multiple scattering of a wave from randomly-distributed impurities and from the resulting destructive interference in the direction of propagation.

Also interactions are well known to induce localization effects, as pointed out by N. F. Mott who was able to explain the anomalous insulator behavior of some materials when electron-electron interactions were included in the band theory. In 1977 P. W. Anderson and N. F. Mott were awarded with the Nobel Prize in Physics for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems (Anderson, 1978; Mott, 1978). Following these pioneering works, a strong theoretical effort has been devoted in the last decades to investigate the combined role of disorder and interactions in the superfluid-insulator transition observed in many condensed-matter systems, such as ^4He adsorbed on porous media (Crowell et al., 1995), thin superconducting films (Goldman and Marković, 1998), arrays of Josephson junctions (Van der Zant et al., 1992) and high-temperature superconductors (Jiang et al., 1994; Budhani et al., 1994).

Ultracold atoms in optical lattices (Morsch and Oberthaler, 2006) represent an extremely powerful tool for engineering simple quantum systems with a broad tunability of the parameters, thus serving as “quantum simulators” (Feynman, 1982) to reproduce the physics of different systems. The striking advantage offered by such atomic systems resides in the unprecedented possibility to work with perfectly isolated samples at quasi-zero temperature and to have experimental control on most of the Hamiltonian parameters, e.g. the lattice depth or the strength of the atom-atom interactions, that can be precisely tuned even in real-time. One spectacular demonstration of this opportunity has been given by the observation of the superfluid (SF) to Mott insulator (MI) transition in a 3D optical lattice (Greiner et al., 2002), which pioneered the investigation of strongly quantum correlated regimes with ultracold atoms (Bloch et al., 2007).

A natural extension of these experiments is the realization of disordered systems using ultracold atoms in optical potentials. In this paper we will review the recent progresses in this field, that was experimentally initiated in 2004 with the first investigation of atomic Bose-Einstein condensates in disordered potentials. Different possibilities can be followed to produce disordered ultracold atomic systems. Disordered or quasi-disordered potentials can be created optically by using speckle patterns (Lye et al., 2005) or multi-chromatic incommensurate optical lattices (Fallani et al., 2007). These methods allow the

production of disordered potentials in which both the spectral properties and the amount of disorder are known with very good accuracy and can be easily controlled. In addition to the optical way, disordered systems could also be created by using atomic mixtures (Gavish and Castin, 2005) or inhomogeneous magnetic fields (Gimperlein et al., 2005; Courteille et al., 2006). We will review these different possibilities together with the illustration of the diverse interaction regimes that can be investigated. The first experimental results obtained with ultracold bosons in disordered potentials will be presented, discussing the state of the art of this newborn field and the perspectives for future breakthroughs.

2 How to produce a disordered potential

In this section we will present different experimental approaches to the production of disordered potentials for neutral atoms. We will mostly focus on two methods allowing the production of complex optical potentials: speckles patterns and multichromatic lattices.

2.1 Speckle patterns

The first realization of disordered potentials for cold atoms has been obtained with speckle patterns (Boiron et al., 1999). Speckles are produced whenever light is reflected by a rough surface or transmitted by a diffusive medium (Goodman, 2006). We will mostly consider the case of transmission, sketched in Fig. 1a, and we will refer to the scattering device as a *diffusive plate*. Such device can be modeled as made up of many randomly-distributed impurities by which the illuminating light is scattered. Since the scattering of laser light is mainly a coherent process, the partial waves emerging from the scattering interfere and produce a complex distribution of light, called a *speckle pattern*, an example of which is shown in Fig. 1b. This disordered distribution of light can be imaged onto the atoms, producing a disordered potential $V(\mathbf{r})$ proportional to the local laser intensity $I(\mathbf{r})$.

In general, if the wavelength of the light is far detuned from the atomic resonance, no absorption is involved and the resulting mechanical effect can be described by a potential energy of the form

$$V(\mathbf{r}) = \frac{3\pi c^2}{2\omega_0^3} \left(\frac{\Gamma}{\Delta} \right) I(\mathbf{r}) , \quad (1)$$

where c is the speed of light, ω_0 is the frequency of the atomic resonance, Γ

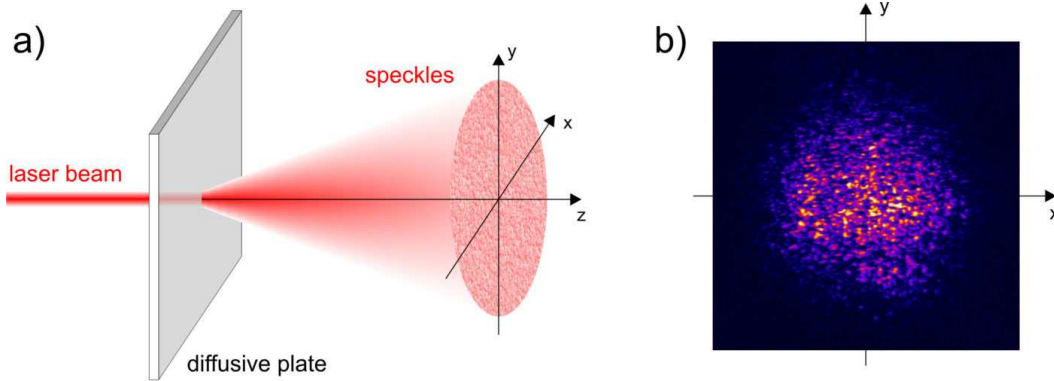


Fig. 1. Production of speckle patterns. a) A laser beam is shone through a diffusive plate and the resulting speckle pattern is then imaged onto the BEC. b) Intensity distribution of a typical speckle pattern recorded with a CCD camera.

its radiative linewidth, $\Delta = \omega - \omega_0$ the detuning, and $I(\mathbf{r})$ the intensity distribution. This potential is often called *dipole potential* (Grimm et al., 2000). It is worth noting that the sign of this potential depends on Δ , which is the only quantity which can take either positive or negative values. In particular, when $\Delta < 0$ (*red detuning*) $V(\mathbf{r})$ is negative, hence maxima of light intensity correspond to potential minima: atoms will move towards higher-intensity regions. Instead, when $\Delta > 0$ (*blue detuning*) $V(\mathbf{r})$ is positive, hence maxima of light intensity correspond to potential maxima: atoms will move towards lower-intensity regions.

Speckle patterns represent a valuable way to produce a disordered potential in a controlled way. The possibility to accurately measure the statistical and correlation properties of the disordered potential comes from the fact that the intensity of the speckle pattern can be directly recorded by a CCD camera (typically the same one used to image the BEC atoms). In Fig. 2a we show the cross section of a typical speckle pattern used at LENS for the first investigation of disordered Bose-Einstein condensates (Lye et al., 2005). Among the different quantities characterizing the properties of the speckle field, one can define an *average speckle height* V_S . Different definitions are used in literature, however one of the most used in the context of BEC experiments corresponds to taking twice the standard deviation of the speckle potential $V(x)$ (supposed one-dimensional) around its mean value \bar{V} (Lye et al., 2005):

$$V_S = 2 \left[\frac{1}{L} \int_{-L/2}^{L/2} (V(x) - \bar{V})^2 dx \right]^{1/2}. \quad (2)$$

An even more important quantity, as we shall see in the following, is the *auto-correlation length* σ , giving information on the speckle grain size (Goodman,

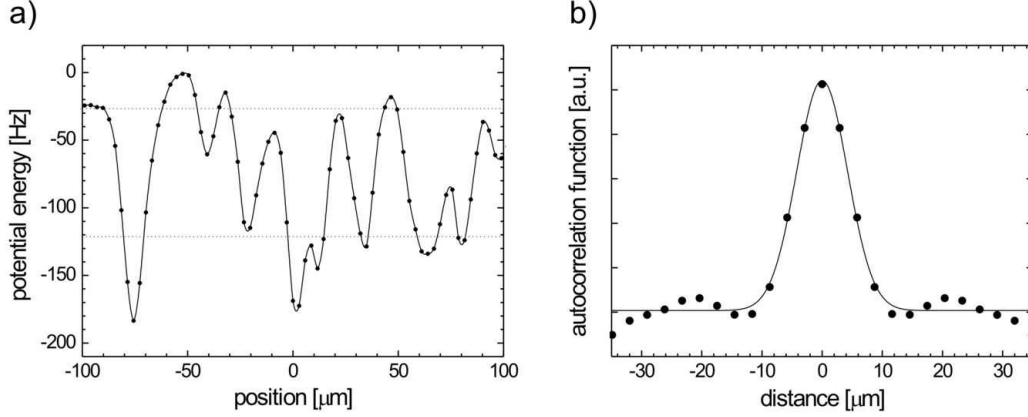


Fig. 2. Production of speckle patterns. a) Cross section of a typical speckle potential (the energy difference between the horizontal lines is the average speckle height V_S). b) Autocorrelation integral of the same potential.

2006). This quantity is defined as the rms width of the autocorrelation integral $G(d)$ of the speckle potential

$$G(d) = \int_{-L/2}^{L/2} V(x)V(x+d)dx \approx e^{-\frac{d^2}{2\sigma^2}}, \quad (3)$$

an example of which is shown in Fig. 2b. The autocorrelation length σ depends on the wavelength of the light, on the nature of the diffusive medium producing the speckle pattern and, most importantly, on the optical resolution of the lens system used to image the speckle pattern onto the atoms. As a matter of fact, the typically determined autocorrelation length is set by the diffraction limit spot size of the imaging system. A detailed description of the speckle potential is given in Clément et al. (2006), where the statistical properties of the speckle field are discussed both from a theoretical point of view and with the introduction of experimental methods which allow their precise determination.

The random potential produced by a speckle pattern is static. This means that the atoms experience just one realization of disorder, which can be reproduced in the same way from one experiment to another. However, shifting the position of the diffuser leads to a different realization of the speckle pattern that preserves the same spectral and statistical properties. Thus, averages on multiple realizations of disorder can be achieved in a simple way.

Before concluding this section, we note that speckles are intrinsically two-dimensional in the plane perpendicular to the propagation axis. Actually, a speckle pattern also varies along the direction of propagation of the light. How-

ever, the typical correlation size along this direction is much larger. Nonetheless, speckle potentials with different dimensionality can be produced: 1D speckles can be produced by using cylindrical lenses stretching the speckle pattern along one direction, while 3D speckles could be obtained by adding speckle patterns coming from different directions.

2.2 Multichromatic lattices

As we have seen in the previous section, speckle patterns are a powerful and easy-to-implement method to produce random potentials. We have pointed out that a crucial parameter of such potentials is the autocorrelation length σ , which gives an estimate of the minimum length scale below which the potential loses its random nature and becomes correlated. Typically, this length is connected with the diffraction limit dimension at which optical speckles are imaged onto the atomic sample. For this reason, the random potential produced by speckles is often too coarse-grained (with σ of the order of several microns), unless one builds a dedicated setup to overcome the usual optical access restrictions. Recently, progresses in the realization of speckle potentials with autocorrelation length below $1\text{ }\mu\text{m}$ have been achieved in the experimental groups of A. Aspect (Clément et al., 2006) and DeMarco (2007).

On the other side, having in mind many years of exciting physics with cold atoms in optical lattices, we know that optical standing waves can be easily created providing spatial periodicities that can also be smaller than half a micron (roughly speaking, one order of magnitude smaller than the autocorrelation length of the speckle pattern shown in Fig. 2). This suggests the idea that, by combining several optical standing waves with different non-commensurate spacings, it is possible to produce complex potentials with very small “grain size”.

The simplest example is given by a bichromatic lattice resulting from the addition of two lattices with incommensurate wavelengths. Bose-Einstein condensates in potentials of this kind have been first investigated in experiments at LENS (Fallani et al., 2007), where a main lattice with wavelength λ_1 was perturbed by a weaker secondary lattice with wavelength λ_2 , as sketched in Fig. 3. The resulting potential can be written in the form

$$V(x) = s_1 E_{R1} \cos^2(k_1 x) + s_2 E_{R2} \cos^2(k_2 x) \quad (4)$$

where $k_1 = 2\pi/\lambda_1$ and $k_2 = 2\pi/\lambda_2$ are the lattice wavenumbers and s_1 and s_2 are adimensional numbers indicating the heights of the two lattices in units of the recoil energies $E_{R1} = \hbar^2/(2m\lambda_1^2)$ and $E_{R2} = \hbar^2/(2m\lambda_2^2)$, respectively. In the limit $s_2 \ll s_1$ the height of the optical barriers is roughly constant across

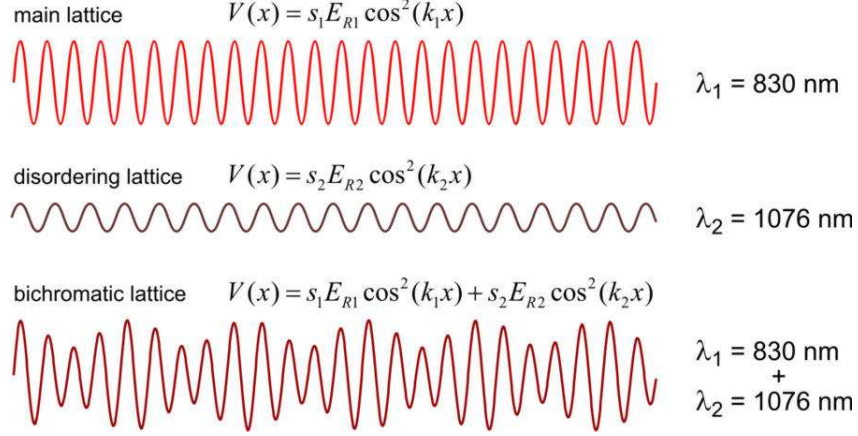


Fig. 3. A bichromatic optical lattice. The discrete translational invariance of the main lattice is perturbed by the addition of a secondary lattice with incommensurate wavelength.

the whole lattice and it is possible to define a tunneling rate J which only depends on the main lattice height s_1 . In this limit, the effect of the secondary lattice reduces to an inhomogeneous and non-periodic shift of the potential energy at the bottom of the lattice wells (see Sec. 3.2.2 and Sec. 4).

As we will further discuss in the following, bichromatic incommensurate lattices are not truly disordered potentials. They differ from both purely random potentials and speckle potentials, which exhibit different statistical and correlation properties. Strictly speaking, they are *quasiperiodic* potentials, since their spectrum is made up of a set of discrete frequencies. However, because of the lack of any translational invariance, they can be used to investigate the physics of finite-sized disordered systems and study the emergence of quantum localization effects, as we shall see in Sec. 3.2.2.

We note that, since one always deals with finite-sized atomic samples, the notion of incommensurability (i.e. the wavelength ratio being an irrational number) is a rather sophistic concept, and should be substituted with a more practical definition. From an experimental point of view, since the lattice wavelengths are known with finite precision, the measurement of the ratio λ_2/λ_1 always gives a rational number. From a theoretical point of view, it is important to consider that the finite size of the systems under investigation releases the constraints on the incommensurability: even a periodic potential (resulting from a commensurate ratio) does not show any periodicity if the system size is smaller than the period. The bichromatic lattice is thus *effectively* incommensurate provided that the ratio between the wavelengths is far from a ratio between *simple* integer numbers. More precisely, a bichromatic lattice can be considered incommensurate whenever the resulting periodicity (if any) is larger than the system size.

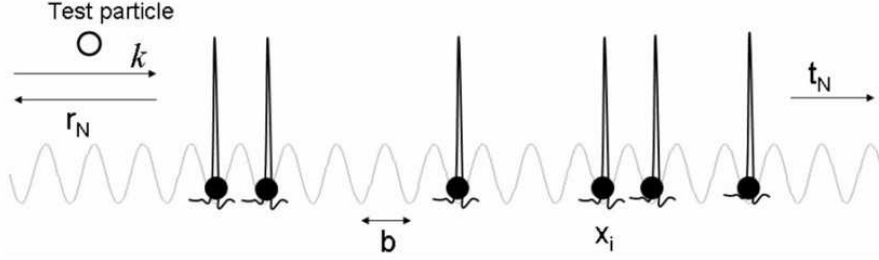


Fig. 4. Disordered potential produced by the interaction of the atoms with a different species. An incoming test particle (white) experiences the collisional potential produced by randomly distributed (black) atoms trapped in an optical lattice. Taken from Gavish and Castin (2005).

2.3 Other methods

In solids disorder is often caused by the presence of impurities, i.e. atoms of a different kind that randomly occupy the sites of the crystalline lattice where atoms of different species were expected. This kind of disorder can be simulated also in cold gases by using a mixture of two different atomic species, as originally proposed in Gavish and Castin (2005). The atoms of one of the two species are trapped in the sites of a deep optical lattice. If the filling factor (i.e. the average number of atoms per site) is less than unity, only some of the sites will be occupied by one atom and the other ones will be empty, as schematically shown in Fig. 4. The atoms of the other species, that could be weakly affected by the presence of the lattice, feel the collisional interaction with the randomly-distributed atoms of the first species. This kind of disorder is spectrally different from both the speckle and bichromatic potentials, since it is a binary kind of disorder (yes/no) on top of a periodic backbone. In Gavish and Castin (2005) the authors have theoretically investigated the possibility to study 1D Anderson localization of matter waves with this system and this work has then been extended in 3D in Massignan and Castin (2006). This scheme has not been experimentally realized yet, although first experiments with binary ultracold mixtures have been performed (see Sec. 4.2).

Another way to introduce disorder in the system has been proposed in Gimperlein et al. (2005) by using inhomogeneous magnetic fields, e.g. by exploiting the magnetic field fluctuations in the proximity of a microtrap caused by imperfections in the chip fabrication (Wang et al., 2004). If the bias magnetic field is kept close to a Feshbach resonance (Inouye et al., 1998), small field fluctuations on top of it produce spatial fluctuations in the scattering length characterizing the interactions between the atoms. Therefore, this technique allows to introduce disorder on the atom-atom interaction strength, rather than on the external

potential. In Gimperlein et al. (2005) the phase diagram of interacting bosons in the presence of such disorder has been derived, evidencing novel features with respect to the phase diagram with disorder in the external potential (that will be presented in Sec. 4).

3 Weakly interacting regime

One of the most fascinating phenomena characterizing the transport of waves in random systems is *Anderson localization*. This effect takes its name after the seminal work of P. W. Anderson in 1958, who identified the fundamental role of disorder in the metal-insulator transition observed in solid state systems (Anderson, 1958). Anderson first formulated his localization theory for a simple model of particles hopping on a lattice with random on-site energies, arguing that above a critical disorder amplitude the quantum states had to change from extended to spatially localized. This intuition, together with the mathematical tools developed to describe the localization transition, led to the award of the Nobel Prize in Physics in 1977 (Anderson, 1978).

In the following decades, however, it was realized that Anderson localization is a much more general phenomenon, holding for propagation of generic linear waves in disordered media. Indeed, it has been observed for sound waves and light waves (Wiersma et al., 1997; Schwartz et al., 2007), whereas a direct observation for matter waves has not yet been possible.

In the language of wave propagation, Anderson localization arises because of interference effects in the scattering of a wave by disordered defects. When studying wave propagation in disordered systems, different localization regimes can be identified. A precursor effect of Anderson localization is *weak localization*, which arises from interference effects in multiple scattering events: an example of weak localization is given by *coherent backscattering*, i.e. the enhanced probability of backdiffusion for light incident on a disordered sample, owing to the interference between the forward and backward scattering paths (Wiersma et al., 1995). In the strong scattering limit $kl \simeq 1$ (with k wavevector and l mean free path between scattering events), these interferences can add up to completely halt the waves inside the random medium, resulting in *strong localization*, or Anderson localization.

Anderson localized states are characterized by the typical exponential decay of their tails in the space distribution. In the case of light, this means that the intensity is an exponentially decreasing function of the distance travelled in the disordered medium. For quantum-mechanical wavefunctions, this means

that a localized state $\Psi(x)$ can be written as

$$\Psi(x) \sim \exp\left(-\frac{x}{\zeta}\right), \quad (5)$$

where ζ is the *localization length*. Generally speaking, the stronger is the disorder the smaller is the localization length.

In the physics of Anderson localization an important role is played by the dimensionality of the system. After the first Anderson's conjecture, scaling arguments have been proposed which predict different scenarios with changing dimensionality d of the system (Abrahams et al., 1979). For $d < 2$ all the states are localized. For $d > 2$ a localization transition exists, with a mobility edge separating extended states for weak disorder from localized states above a critical value. The 2D case is marginal, since the states are localized for any amount of disorder as in 1D, but the localization length at weak disorder can be exponentially large.

A Bose-Einstein condensate is characterized by long-range coherence and can be described with a classical order parameter which corresponds to the wavefunction of the Bose-condensed atoms. In the noninteracting case all the atoms are described by the same single-particle wavefunction which obeys the Schrödinger equation

$$i\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\mathbf{r})\Psi. \quad (6)$$

In the presence of disorder, this wavefunction can be Anderson localized. Bose-Einstein condensates represent an appealing system where it is possible to directly study the effect of localization. By using the techniques described in the previous section, one is able to create disordered potentials in an extremely controlled way, knowing precisely the kind and amount of disorder. Furthermore, the wavefunction (more precisely, the squared modulus of it) can be directly observed by imaging the condensate with a CCD camera. As a result, the typical exponential tails of Anderson localized states could be observed (at least in principle, if the imaging resolution and sensitivity are good enough), allowing the detection of localization.

An extremely interesting, and still open, problem regards the effect of interactions on localization. Originally, Anderson formulated his theory for noninteracting quantum particles. If one considers real interacting particles, however, the scenario could be significantly different. In the case of electrons, repulsive long-range interactions are present due to the Coulomb electric force. In the case of ultracold neutral atoms long-range dipolar interactions can be nearly always neglected and the dominant interaction mechanism is represented by

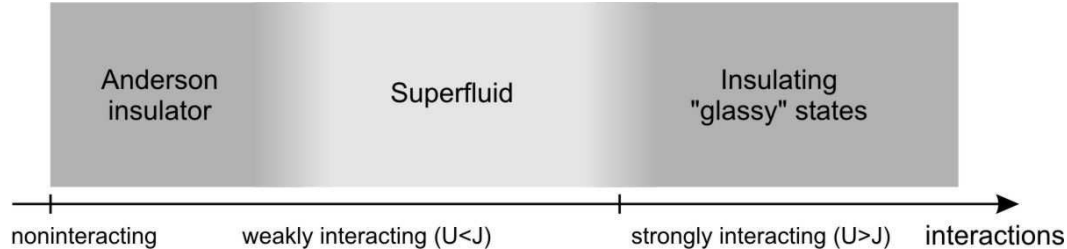


Fig. 5. Pictorial representation of the different interaction regimes for a bosonic gas in the presence of disorder.

s -wave elastic collisions. These short-range interactions can be either attractive or repulsive, although only repulsive interactions allow for the existence of stable BECs with arbitrarily large number of atoms (Dalfovo et al., 1999). When the BEC density is sufficiently small, as it happens in many experimental situations, the effect of the collisional forces can be described within a mean-field approach by adding a nonlinear term in Eq. (6), which becomes the well-known Gross-Pitaevskii equation (GPE) (Dalfovo et al., 1999):

$$i\hbar \frac{d\Psi}{dt} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\mathbf{r})\Psi + g|\Psi|^2\Psi. \quad (7)$$

The interaction strength, described by $g = 4\pi\hbar^2 a/m$, is parametrized as a function of one single scalar parameter a , which takes the name of *scattering length*. A similar kind of cubic nonlinearity is present also in the Maxwell equations describing the propagation of light in a nonlinear optical medium where the index of refraction depends on the light intensity (Kerr effect). This term is responsible for many effects of nonlinear dynamics, such as solitonic propagation (Burger et al., 1999), four-wave mixing (Deng et al., 1999) and instabilities (Wu and Niu, 2001; Fallani et al., 2004). The presence of interactions can heavily affect the physics of localization, which is intrinsically a single-particle effect, holding for linear waves. From a naive point of view, negative nonlinearities ($a < 0$, arising from attractive interactions between particles, or self-focusing behavior of the wave) could play in favor of localization. On the contrary, positive nonlinearities ($a > 0$, induced by repulsive interactions, or self-defocusing behavior) are expected to play against localization, making the problem much more interesting to study, both theoretically and experimentally.

The interplay between disorder and interactions in the physics of localization has been the object of a very intense theoretical investigation. It was soon realized that repulsive interactions can compete with disorder and eventually destroy the localization. In strongly interacting systems, however, different regimes can be achieved and new quantum phases can be reached in which interactions and disorder co-operate in localizing the system in glassy states

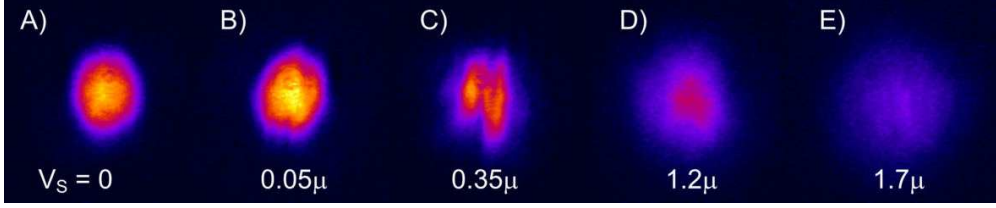


Fig. 6. Ground state of a BEC in a harmonic + disordered potential. Absorption images of the atomic density distribution following a time-of-flight after the release from the confining potential. The numbers in the bottom indicate the average speckle height V_S in units of the BEC chemical potential $\mu \simeq 1$ kHz in the harmonic trap. Adapted from Lye et al. (2005).

(see Sec. 4 for a discussion of these phases). An oversimplified picture of the different regimes for an interacting bosonic gas in a disordered potential is sketched in Fig. 5. Real phase diagrams, of course, are much more complicated than this pictorial representation, the details depending e.g. on the kind of disorder and on the dimensionality of the system. Actually, there are still many open questions to be solved, which the experimental study of interacting Bose-Einstein condensates in disordered potentials could address. As a matter of fact, these systems offer the advantage of a broad tunability of the Hamiltonian parameters, including the kind and amount of the disorder (as seen in the previous section) and the interaction strength between the atoms (e.g. by using Feshbach resonances (Inouye et al., 1998)).

3.1 A Bose-Einstein condensate in a disordered potential

3.1.1 Static properties

A natural starting point to gather information on the behavior of the BEC in the disordered potential is the shape of the atomic density distribution after release from the confining potential. This time-of-flight detection technique has been used since the first experimental realization of BEC (Anderson et al., 2005) as a precious tool to study its ground state properties. We start considering the case of disordered potentials created with optical speckles, first investigated with ^{87}Rb in Lye et al. (2005).

Basically, one can observe three different regimes, depending on the ratio between the speckle height V_S and the BEC chemical potential μ . For very small optical potentials $V_S \lesssim 0.1\mu$ one does not observe any significant deviation from the ordinary Thomas-Fermi shape of the BEC expanding from the harmonic trap (fig. 6A,B). For higher speckle heights $0.1\mu \lesssim V_S \lesssim \mu$ one observes that the density distribution is strongly modified by the appearance of complex

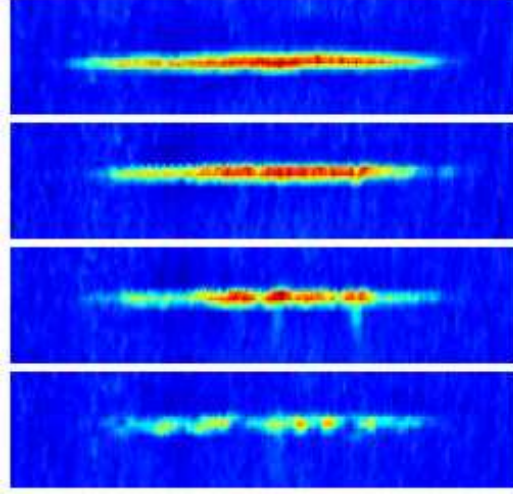


Fig. 7. Ground state of a BEC in a harmonic + disordered potential. *In-situ* images of the atomic density distribution. The speckle potential V_S is increasing from top to bottom: 0, 0.3μ , 0.5μ , 1.3μ . Taken from Chen et al. (2008).

structures in the form of elongated stripes (fig. 6C). Finally, further increasing the speckle height to $V_S \gtrsim \mu$, the expanded density profile ceases to be characterized by stripes and one can detect only a broad unstructured gaussian distribution (Fig. 6D,E).

The appearance of density modulations in the regime of weak disorder has recently attracted a large interest. The structures observed after expansion could arise either from real in-trap density modulations or from phase fluctuations converted into density modulations after time-of-flight. More recent experimental works (Chen et al., 2008; Clément et al., 2008) systematically investigated such structures, measuring the fringes visibility as a function of the speckle height and evidencing that the fringes pattern is stable for the same realization of disorder. This observation, together with the comparison with GPE simulations (Clément et al., 2008), suggests that for the actual experimental parameters the most plausible scenario is the one in which small in-trap density fluctuations are amplified during the time-of-flight. The problem is studied in detail in Clément et al. (2008) with a thorough analysis of the mechanisms involved during the BEC expansion.

Further increasing the intensity of the disordered potential to $V_S \gtrsim \mu$ the condensate is split up in many condensates localized in the randomly-spaced minima of the speckle potential. The absence of interference structures in the observed density distribution is due to the fact that the spacing between different condensates is not uniform and gives rise to an interference pattern that, averaged over the optical resolution of the system, is almost flat. This *fragmentation* scenario has been confirmed in Chen et al. (2008) by using direct

in-situ imaging of the trapped ^7Li atoms (see Fig. 7).

3.1.2 Collective excitations

Low-energy collective excitations of the BEC have been studied in the presence of a weak speckle disorder producing a corrugation of the harmonic trap potential. Since the first production of BECs, frequency measurements of collective modes have provided precious information for the identification of superfluidity and, more in general, for the characterization of quantum fluids. In Lye et al. (2005) the frequency and the damping rate of the dipole and quadrupole modes of a BEC in a speckle potential have been measured as a function of the strength of disorder. With the term of “dipole mode” one usually refers to rigid center-of-mass oscillations of the BEC in the parabolic trap, while the “quadrupole mode” for a highly anisotropic trap (or “axial breathing mode”) indicates a shape oscillation in which the center-of-mass does not move, but the size of the BEC along its long axis is periodically changing in time (Dalfovo et al., 1999). In Lye et al. (2005) a damping of both modes has been observed and measured as a function of the strength of the speckle potential.

Damping of the dipole mode has been recently observed also in Chen et al. (2008), in which the study of the different dynamic regimes have been accompanied by *in-situ* detection of the atomic density distribution. Indeed, Chen et al. (2008) demonstrated that the halting of the center-of-mass motion for strong disorder has to be connected with the creation of a fragmented BEC. In this regime different condensates are trapped in the different speckle potential wells and no global phase coherence is present, due to the extremely long tunneling times between different fragments.

Lye et al. (2005) measured also a frequency shift of the quadrupole mode. The frequency of the quadrupole mode is particularly important since its value depends not only on the strength of the parabolic potential but also on the Bose-Einstein equation of state, hence it depends on the nature of the system, whether it is superfluid or not. The frequency changes measured in Lye et al. (2005), however, just reflected the change in the effective potential curvature induced by the corrugation produced by the disordered potential. The problem has been addressed theoretically in Modugno (2006) by numerical solution of the Gross-Pitaevskii equation combined with a sum-rules approach, confirming the frequency shift as an effect due to the change in the effective trap frequency.

The effect of the speckle potential produced in these experiments is mostly classical, and it does not really produce a change in the nature of the quantum fluid. We shall discuss more about this point in the next section, evidencing how the correlation length of the potential plays a crucial role for the observa-

tion of truly disordered-induced localization effects. In particular, concerning the measurements of collective excitations, the presence of disorder with short correlation length can modify the superfluid equation of state leading to non-trivial frequency shifts, as recently studied in Falco et al. (2007).

3.2 *The quest for Anderson localization*

In this section we will discuss the state-of-the-art of the experiments aiming to observe Anderson-like localization for Bose-Einstein condensates propagating in disordered optical potentials.

3.2.1 *Localization in a speckle potential*

Out of the condensed matter systems for which it has been originally proposed, Anderson localization has been widely searched, and eventually demonstrated, in classical wave propagation experiments (Wiersma et al., 1997; Schwartz et al., 2007). In this kind of experiments, an electromagnetic wave undergoes multiple scattering from the randomly-distributed scatterers of the disordered medium. Strong (Anderson) localization sets in when the multiple scattered waves interfere destructively in the propagation direction and localized states become populated. According to the Ioffe-Regel criterion (Ioffe and Regel, 1960) this happens when the mean free path of the wave becomes as small as its wavelength.

Experiments performed in 2005 at LENS (Florence) and in the group of A. Aspect at Institut d’Optique (Orsay) aimed to realize such scattering configuration with Bose-Einstein condensates propagating in disordered optical potentials produced with speckle patterns (Clément et al., 2005; Fort et al., 2005). The idea behind these two works was quite similar: an initially trapped Bose-Einstein condensate of ^{87}Rb was left free to expand in a one-dimensional disordered waveguide. In Clément et al. (2005) this waveguide was produced by a highly elongated magnetic trap, while in Fort et al. (2005) by a single beam optical trap. The propagation of the condensed matter wave in the waveguide was studied as a function of the height of the disordered speckle potential.

In Fig. 8, taken from Fort et al. (2005), the density distribution of the condensate, imaged *in situ* after a fixed expansion time in the optical waveguide, is shown for different speckle potential heights (ranging from $V_S = 0$ to $V_S = 0.7\mu$, with μ the BEC chemical potential) together with the picture of the actual speckle field used. Without speckles the condensate freely expands, while in the presence of the speckles both the expansion and the center-of-mass motion (induced by a small acceleration along the waveguide) start to

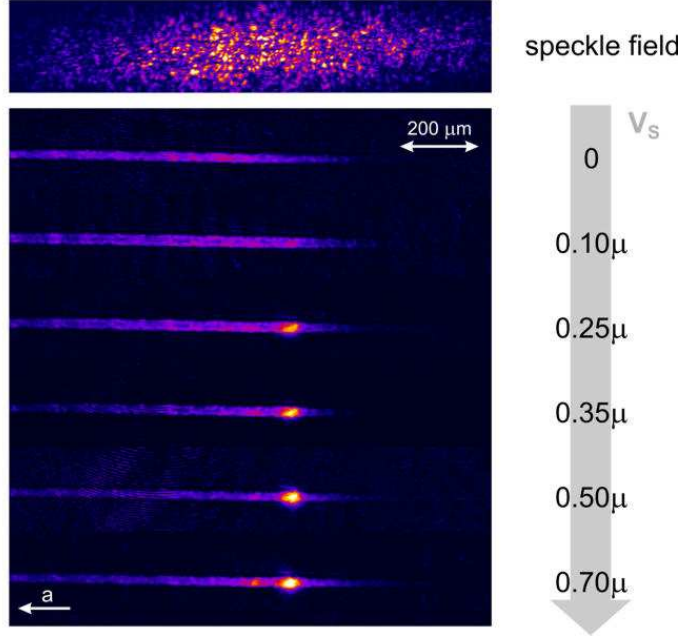


Fig. 8. Expansion of a BEC in a disordered optical guide. Top) Intensity profile of the speckle field used in the experiment. Bottom) Density profiles of the condensate after expansion in the disordered optical guide for different speckle heights V_S , here expressed in units of the BEC chemical potential $\mu = 2.5$ kHz in the initial trap. Adapted from Fort et al. (2005).

be suppressed for $V_S \gtrsim 0.3\mu$. A closer look shows that actually two different components can be distinguished: while a low density cloud expands without stopping, a few localized density peaks become observable when increasing the speckle height. In Fig. 9, taken from Clément et al. (2005), the rms size of the BEC expanding in a disordered magnetic waveguide is plotted as a function of time for different heights of the disordered potential: one can clearly see the transition from a diffusive regime in the absence of disorder to a “localization” regime when disorder is present.

Further investigations have demonstrated that this suppressed expansion is not Anderson localization, but a classical localization that can be explained with simple energetic arguments. The expanding condensate is not a monochromatic flux of atoms all moving with the same velocity: since the momentum distribution of the sample has a finite width (mainly caused by the atom-atom repulsive interactions which initially drive the expansion), a low velocity component of the cloud is always present and get trapped in the speckles since it has not sufficient energy to escape the deepest potential wells (in the case of red-detuned speckles, as in Fort et al. (2005)) or to tunnel through the highest potential barriers (in the case of blue-detuned speckles, as in Clément et al. (2005)).

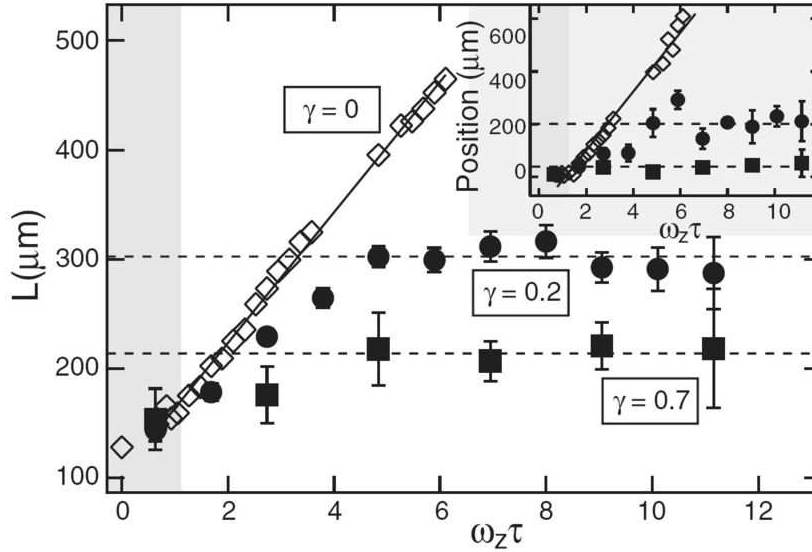


Fig. 9. Expansion of a BEC in a disordered magnetic waveguide. Time evolution of the axial BEC rms size L and center-of-mass position (inset) for various amplitudes γ of the random speckle potential (in units of the BEC chemical potential). Taken from Clément et al. (2005).

Several theoretical works have studied the expansion of an interacting Bose-Einstein condensate in a speckle potential (Clément et al., 2006; Modugno, 2006; Shapiro, 2007; Sanchez-Palencia et al., 2007; Akkermans et al., 2008; Sanchez-Palencia et al., 2008). In particular, in Clément et al. (2005, 2006) and Modugno (2006) it has been shown that the expanded BEC density profile is actually made up of two spatially separated parts. In the center of the cloud interaction energy is dominating over kinetic energy and the BEC density profile exactly follows the shape of the potential, as expected from the Thomas-Fermi approximation for an interacting Bose gas: no Anderson localization is expected to appear in this region. In the wings of the cloud the density is much smaller and kinetic energy is dominating over interaction energy: here the BEC almost behaves as a noninteracting gas and the density profile has deviations from the Thomas-Fermi approximation. However, numerical studies based on the Gross-Pitaevskii equation evidenced that no Anderson localization is present even in this region.

There are two possible physical reasons impeding the observation of Anderson localization. The first is indeed the presence of interactions: from an intuitive point of view, repulsive interactions between the atoms force them to spread more in space, contrasting localization. The second reason is the finite correlation length of the disorder: even in the absence of disorder, Anderson localization could not be observable because the disordered potential is not “good” enough to produce the scattering strength which is necessary to have

a localization length smaller than the system size. It is well known that, in the pure random case, any infinitesimal amount of disorder leads to localization in 1D (Abrahams et al., 1979), but in finite-sized systems (like a trapped Bose-Einstein condensate) the localization length plays an important role.

The further experiments reported of Fort et al. (2005) have evidenced that, apart from the problem of interactions, the typical speckle potentials employed so far in the experiments were not “fine-grained” enough to produce quantum reflection/transmission, which is at the basis of 1D Anderson localization. This has been observed by studying the collision of a BEC with a potential defect created with a tightly focused laser beam, that mimics the effects of one single speckle grain. The absence of quantum reflection from the potential well created with this optical (red-detuned) defect indicated that, in order to have quantum scattering, one should use much steeper potentials, i.e. speckle potentials in which the autocorrelation length σ is smaller (the typical correlation length of the speckles used in Fort et al. (2005) and Clément et al. (2005) was $5\text{ }\mu\text{m}$).

This problem has been theoretically addressed in Modugno (2006), where the coefficients of quantum reflection by a potential well and of quantum transmission from a potential barrier have been calculated as a function of the potential steepness and of the velocity of the incident matter waves. Of course, quantum reflection/transmission is fundamental in the case of 1D localization, in which scattering just happens along a line: hence, for multiple scattering to appear, only a fraction of the incident wave has to be reflected/transmitted. In higher dimensions interference due to multiple scattering could happen also for classical reflection from potential hills, for which the requirements are less stringent. However, in higher dimensions localization itself is more difficult to achieve, owing to the larger localization lengths (if localization is present).

Despite the obstacles discussed above, Anderson localization could be eventually observed in 1D diffusion experiments similar to the ones reported in Clément et al. (2005) and Fort et al. (2005), provided that the speckle autocorrelation length is made small enough. In Sanchez-Palencia et al. (2007) the density profile of the BEC expanding in a weak speckle potential has been analytically worked out, evidencing exponential localization in the dilute tails of the wavefunction, where density is very low and interactions can be neglected. In the case of interacting ^{87}Rb BECs this could happen for very small disorder correlation length, one order of magnitude less than the ones achieved in Clément et al. (2005) and Fort et al. (2005). The crucial parameter, as we shall see in the following section, is the ratio between the disorder correlation length and the *healing length*, which is the length scale associated to the effect of interactions. In Sanchez-Palencia et al. (2007) a mobility edge was also found as a maximum wavevector of the expanding BEC above which localization cannot be observed, which is a peculiar characteristic of speckle

potentials with finite correlation length.

3.2.2 Localization in a bichromatic lattice

Quasiperiodic lattices, introduced in Sec. 2.2, are a particular class of potentials which exhibit properties common to both periodic and disordered systems (Diener et al., 2001). As in the case of periodic lattices, their spectrum show reminiscence of energy bands. On the other hand, owing to the lack of any translational invariance, they support the existence of localized states, which behave very similarly to the ones supported by truly disordered systems (Grepel et al., 1982). Therefore they can be used as a tool to study quantum localization, as a valid alternative to speckle patterns, with the experimental advantage of an effortless production of short length-scale potential fluctuations.

Localization in incommensurate bichromatic potentials is a well known topic. This problem has been studied in detail in the framework of the Harper model (Harper, 1955) and of the 1D tight-binding Aubry-André model (Aubry and André, 1980), which is described by the Hamiltonian

$$\hat{H} = -J \sum_n (|n\rangle\langle n+1| + |n+1\rangle\langle n|) + \Delta \sum_n \cos(2\pi\beta n) |n\rangle\langle n|, \quad (8)$$

where J is the tunneling rate between next-neighboring sites and Δ is the amplitude of the quasiperiodic modulation of the potential energy, being β an irrational number. The Aubry-André model can be experimentally realized when the primary lattice height s_1 is much larger than the secondary lattice height s_2 (see Sec. 2.2). The primary lattice discretizes the system and produces a renormalization of the effective mass $m^* = mE_{R1}/J\pi^2$, while the height of the secondary lattice $s_2 = \Delta/E_{R2}$ is the control parameter which drives the localization transition. As a matter of fact, differently from what happens with a pure disordered potential, in the quasiperiodic case a localization transition exists even in 1D, with a critical value $\Delta \approx 2J$ of the quasidisorder amplitude for producing a localized ground state.

This behavior is illustrated in Fig. 10a, in which we plot the lowest-energy single-particle eigenstate in the incommensurate lattice, as obtained by numerical integration of the 1D Schrödinger equation (6), which holds in the continuum. The figure shows, in grayscale, the squared modulus of the ground state wavefunction as a function of position (horizontal axis) and disordering lattice strength (vertical axis). One clearly sees that for low values of disorder the ground state is an extended state (see Fig. 10c), i.e. the wavefunction extends across the entire lattice (in the limit $\Delta = 0$ one recovers the extended Bloch state describing the ground state in a periodic lattice). Increasing disorder above a threshold value the wavefunction suddenly localizes around few

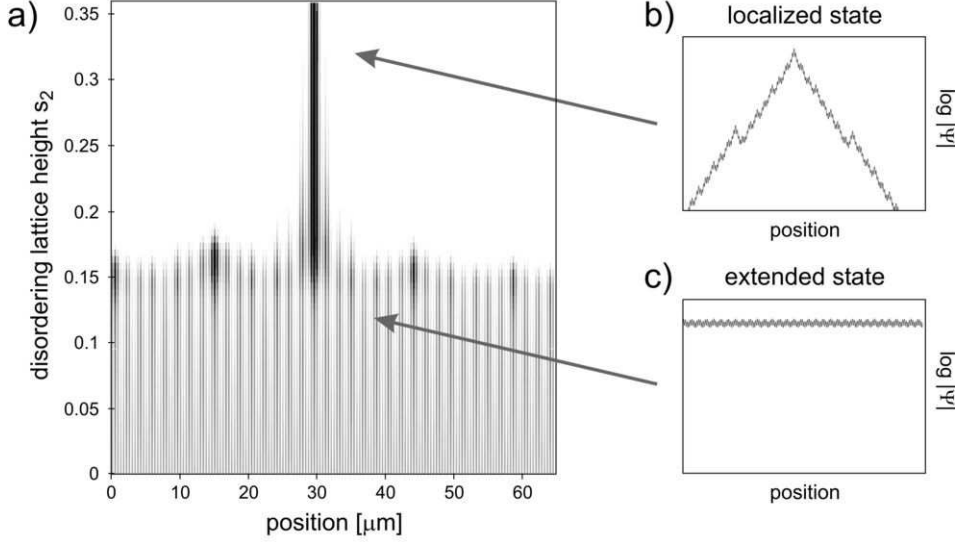


Fig. 10. Transition from extended to localized states in a bichromatic incommensurate potential. a) The square modulus of the ground state wavefunction is plotted in grayscale as a function of position and disordering lattice strength. b,c) Logarithmic plots of the ground state wavefunction below and above the localization transition.

lattice sites, with exponentially decreasing tails, in the same way as in the case of Anderson localized states in δ -correlated disordered potentials (Fig. 10b).

So far, we have shown that the incommensurate lattice supports the existence of localized states for noninteracting particles. What happens when one introduces interactions? This problem has been discussed in Schulte et al. (2005, 2006), where the ground state of the system has been calculated for a three-colour optical lattice. Introducing repulsive interactions between the atoms, the numerical integration of the 1D Gross-Pitaevskii equation shows that the ground state wavefunction becomes a superposition of many single-particle localized states, which add up to form an overall extended state, as shown in Fig. 11 for different interaction strengths. Similar results for a bichromatic lattice have been presented in Lye et al. (2007).

This behavior can be interpreted in terms of a *screening effect* induced by interactions (Sanchez-Palencia, 2006; Schulte et al., 2006). The nonlinear term in the Gross-Pitaevskii equation (7) can be treated as an effective potential cancelling the spectral components of the original potential varying on length scales larger than the *healing length* $\xi = 1/\sqrt{8\pi an}$. This latter quantity is the typical length scale that is associated to the variation of the BEC wavefunction around sharp potential jumps (Dalfovo et al., 1999). More generally, the healing length is the typical length scale below which the condensate wavefunction is able to behave quantum-mechanically. As a consequence, in order to observe localization, the interference effects producing localization should take place

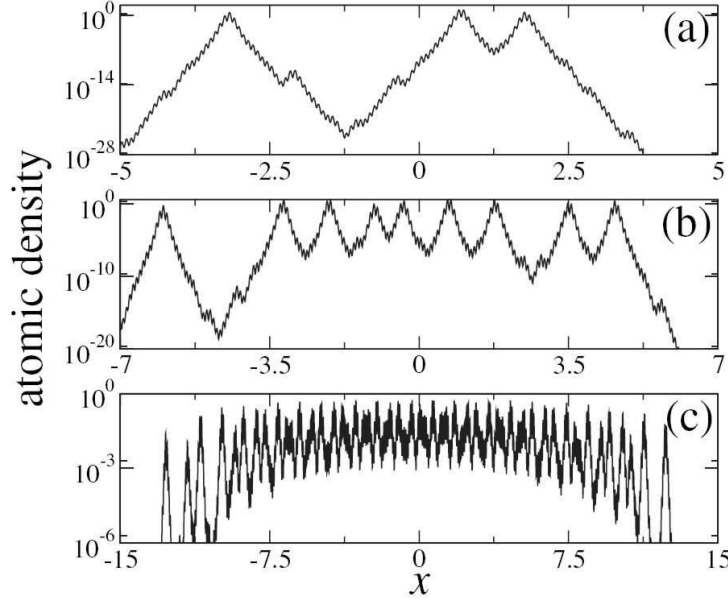


Fig. 11. Ground state of the Gross-Pitaevskii equation for a Bose-Einstein condensate in a three-colour optical lattice for different effective interaction strength $\tilde{g} = 0.2$ (a), $\tilde{g} = 8$ (b), $\tilde{g} = 256$ (c). The latter value corresponds to the interaction strength for a Bose-condensed sample of $N = 10^4$ ^{87}Rb atoms in an elongated harmonic trap with frequencies $\omega_{\perp} = 2\pi \times 40$ Hz and $\omega_{\parallel} = 2\pi \times 4$ Hz. Taken from Schulte et al. (2005).

on a distance smaller than the healing length, otherwise the BEC wavefunction would behave classically over longer distances. The healing length can be made larger by reducing the amount of interactions in the system, that could be achieved either by reducing the scattering length a or by reducing the atom density n . If the healing length is smaller than the disorder localization length ($\xi < \zeta$, see Eq. 5) no localized states can be observed. If the healing length is larger than the localization length, but smaller than the system size ($\zeta < \xi < L$), one could observe a superposition of many localized states. Finally, if the healing length is the largest length scale in the system ($\xi > L > \zeta$) the BEC wavefunction collapses in a single localized state. This crossover is illustrated in Fig. 11, where the ground state in the three-color lattice is plotted for the same lattice heights but different interaction strength (hence same ζ but different ξ): while in panel a) $\xi > \zeta$ and a few localized states can be easily detected with clearly exponentially decreasing tails, in panel c) $\xi < \zeta$ and an overall extended state forms.

The existence of localized states can be probed with transport experiments, similar to those presented in the previous section. In Lye et al. (2007) the transport of an ^{87}Rb BEC has been studied in the presence of a bichromatic incommensurate potential. Localization of the center-of-mass motion has been

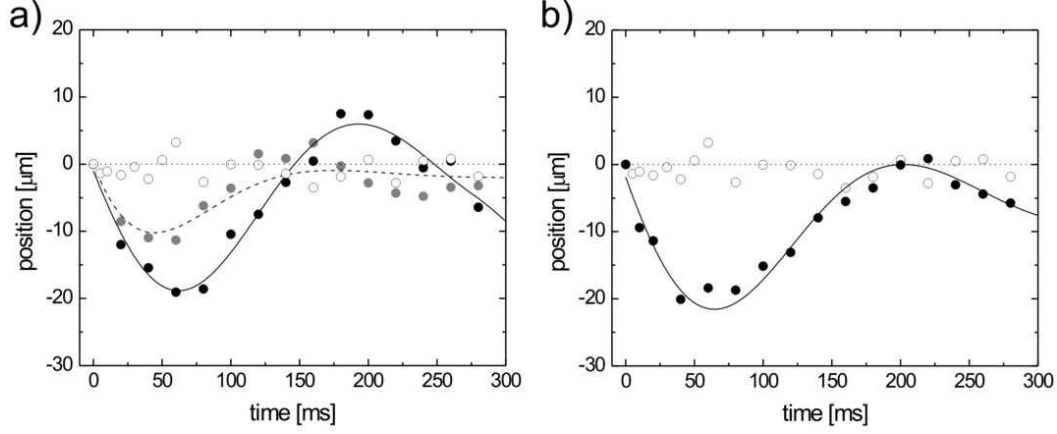


Fig. 12. Oscillations of a BEC in a parabolic + incommensurate two-color lattice. a) Center-of-mass position for $N = 1.5 \times 10^4$ atoms, $s_1 = 10$ and different disordering lattice heights $s_2 = 0.1$ (filled black circles), $s_2 = 0.17$ (filled gray circles), $s_2 = 0.25$ (empty circles). b) Center-of-mass position for $s_1 = 10$, $s_2 = 0.25$ and different atom numbers $N = 1.5 \times 10^4$ (empty circles) and $N = 2 \times 10^5$ (filled circles). Adapted from Lye et al. (2007).

observed, the stronger the smaller is the strength of interactions (tuned by changing the atomic density), as shown in Fig. 12. This density-dependent behavior, with interactions pushing to delocalize the system, is reminiscent of Anderson-like localization. However, in the regime of parameters studied in this work, no simple Anderson-like localization has to be expected, being the eigenstates of the system similar to the state dominated by interactions shown in Fig. 11c. The suppression of the center-of-mass motion shown in Fig. 12 was mainly caused by the strong modulation of the BEC wavefunction on the length scale of the beating between the two lattice periods, which resulted in very low tunneling times across the lattice and, consequently, in an extremely slow dynamics.

Effects of nonlinear dynamics have also been considered as possible mechanisms to damp the motion. As a matter of fact, suppression of transport is expected to appear for mechanisms alike the interaction-induced dynamical instability observed in Cataliotti et al. (2003), Fallani et al. (2004) and Cristiani et al. (2004) for monochromatic optical lattices. In the system studied in these works the interplay between repulsive nonlinearities and band structure resulted in fast-growing excitations dephasing the system and halting the motion. The same effect can also be observed in bichromatic optical lattices, in which a band structure can still be identified (Diener et al., 2001), with a multitude of energy gaps opening in the spectrum and getting denser and denser with increasing height of the secondary incommensurate lattice.

From what discussed above we can draw a preliminary conclusion. Bichro-

matic potentials do allow to solve the problem of the correlation length of the speckles produced in Lye et al. (2005); Clément et al. (2005); Fort et al. (2005); Schulte et al. (2005), which causes the localization length to be too large to be observable. In bichromatic potentials the localization length can easily be smaller, however the presence of too strong interactions still remains and makes it impossible to observe a clear localization of the wavefunction in a few localized states. In order to achieve clear signatures of Anderson localization one needs to work with extremely weakly interacting samples.

3.3 Further directions

From the theoretical point of view, as we have already pointed out, the effect of disorder on the interacting Bose gas is an extremely interesting topic of research. In Lugan et al. (2007a) the ground state of an interacting Bose gas in a disordered potential has been deeply studied. In particular, a Lifshitz glass phase has been introduced characterizing the ground state of the system for weak interactions. Lifshitz states are a particular class of localized states which exhibit “weaker” localization properties than Anderson-localized states, in the sense that they show exponential decay only in the very far tails, while close to the maximum their shape mostly depends on the *local* properties of the potential. This means that they mostly resemble bound states of isolated potential wells or trapped states between barriers, differently from Anderson-localized states, whose shape is determined by *global* properties of the potential, i.e. by the combined effect of many impurities / potential wells. In Fig. 13 we show the phase diagram of the interacting disordered BEC derived in Lugan et al. (2007a) as a function of the BEC chemical potential and of the speckle height. Starting from this Lifshitz glass phase and increasing interactions, a phase of fragmented interacting BECs has been proposed, which is a precursor of the Bose glass phase (see Sec. 4 for further discussion).

The BEC fragmented state has been previously described in Wang et al. (2004). In this work the authors studied the ground state of a BEC in the disordered potential produced by the random imperfections of a magnetic microtrap. This paper was motivated by several experimental observations (Fortágh et al., 2002; Leanhardt et al., 2003; Estève et al., 2004; Jones et al., 2004), in which fragmentation of the BEC at very close distances from the current-carrying wires of the microchip was observed. In the same work (Wang et al., 2004) the BEC dynamics in the disordered potential was also investigated. In particular, the spectral analysis of the sloshing motion after displacement of the confining potential allowed to identify different dynamical regimes: superfluid oscillations, self-trapping and an intermediate chaotic regime.

Recent theoretical works (Bilas and Pavloff, 2006; Lugan et al., 2007b) have

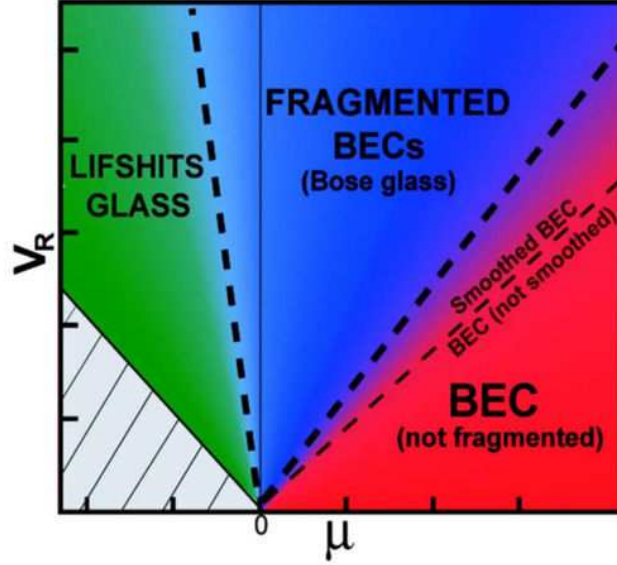


Fig. 13. Phase diagram of an interacting disordered BEC as a function of the chemical potential μ and of the disordered potential height V_R . Taken from Lugan et al. (2007a).

also studied the problem of Anderson localization of excitations in a Bose-Einstein condensate. In the weakly interacting case, BEC excitations are described by the Bogoliubov theory (Dalfovo et al., 1999). By calculating the leading-order many-body corrections to the classical BEC wavefunction, one finds that, in the absence of external potentials, excitations are described by quasiparticles with dispersion relation $\hbar\omega = \sqrt{(\hbar ck)^2 + (\hbar^2 k^2/2m)^2}$, with k wavevector of the excitation and c sound velocity inside the BEC. This spectrum has two distinct regions with different k -dependencies. For $k \ll \xi^{-1}$ (with ξ the healing length) the excitation spectrum is phonon-like and excitations have energy $\omega \approx ck$. For $k \gg \xi^{-1}$ the spectrum is particle-like and the energy of the excitations is $\omega \approx \omega_0 + \hbar k^2/2m$, where $\omega_0 = mc^2/\hbar$ is an energy shift due to interactions. Bilas and Pavloff (2006) have shown that, in the presence of a white-noise random potential, excitations can undergo Anderson localization, almost in the same way as the whole BEC wavefunction can undergo. Further detailed studies have been carried out in Lugan et al. (2007b) in the more realistic case of correlated disorder, showing that the localization length (and, correspondingly, the possibility to observe localization in finite-sized BECs) crucially depends on the correlation length of the disorder.

The effect of disorder on the coherent BEC dynamics can be observed also on the dephasing of Bloch oscillations which is expected to appear when a disordered or quasi-disordered potential is superimposed on a tilted optical lattice (Sanchez-Palencia and Santos, 2005; Schulte et al., 2008). Bloch oscillations are the coherent oscillations of a wavepacket in a periodic potential when a

constant force is applied. This phenomenon has been observed for the first time with ultracold atoms in optical lattices (Raizen et al., 1997) because of the much longer coherence times than the ones achievable for electrons moving in real solid-state lattices, where defects and impurities strongly dephase the system in a time much shorter than the oscillation period. Also interactions lead to dephasing, as evidenced in Morsch et al. (2001) when Bloch oscillations were observed for the first time in a Bose-Einstein condensate. Later it was demonstrated that this interaction-induced dephasing can be controlled and eventually cancelled by tuning the interaction strength with Feshbach resonances, as recently demonstrated in Gustavsson et al. (2008) and Fattori et al. (2008), or by using ultracold fermionic samples (Roati et al., 2004), for which interactions are forbidden by the Pauli principle. Noninteracting particles in perfectly periodic optical lattices perform undamped Bloch oscillations and, thanks to this possibility, they can be used as microscopic probes for high-precision measurements of forces at small distances (Carusotto et al., 2005). Starting from this ideal situation and adding disorder on top of the periodic lattice, one can quantitatively study the dephasing induced by disorder in a controlled way, as first theoretically studied in Schulte et al. (2008) and then recently investigated experimentally in Drenkelforth et al. (2008).

Generally speaking, disorder leads to the disruption of coherent effects. Quite interestingly, however, under certain conditions disorder can induce a spontaneous ordering of the system. This effect, known as *random-field induced order*, has been originally studied in the context of classical spin models, which in the presence of disorder may exhibit a magnetization higher than in the ordered case (Wehr et al., 2006; Sen De et al., 2007). Two-component Bose-Einstein condensates in the presence of a random Raman coupling between the two states can be used to study this class of effects, as recently proposed in Niederberger et al. (2008).

3.4 *Anderson localization: the state of the art*

Starting from the first experiments with speckle potentials (Lye et al., 2005; Fort et al., 2005; Clément et al., 2005; Schulte et al., 2005), the quest for Anderson localization in Bose-Einstein condensates has been a strongly active direction of research. On one side experimental groups have focused on producing disordered potentials on thinner length scales (Clément et al., 2006; DeMarco, 2007), in order to increase the amplitudes of quantum scattering and decrease the attainable localization lengths. On the other side, the challenge is to reduce atom-atom interactions in order to make localization observable. Once this is obtained, it will be even more interesting to study the effect of adding a controlled amount of interactions. For this purpose it can be strongly helpful to take advantage of Feshbach resonances to tune the scatter-

ing length a , which is the key parameter defining the strength of interactions $g = 4\pi\hbar^2 a/m$ (see Sec. 3). In this perspective, the choice of the element under investigation is crucial. The first experiments performed with BECs in disordered potentials (Lye et al., 2005; Fort et al., 2005; Clément et al., 2005; Schulte et al., 2005) have focused on ^{87}Rb , which is a quite convenient element for the implementation of cooling schemes, but has the disadvantage of having a quite large scattering length $a \simeq 100a_0$ (with a_0 the Bohr radius) and no favorable Feshbach resonances at convenient magnetic fields (Marte et al., 2002). A much easier tuning of atom-atom interactions could be provided by different elements, such as ^7Li (which has been already studied in combination with laser speckles in Chen et al. (2008)) or ^{39}K , studied by Roati et al. (2007).

While this review was being completed (march 2008) two experiments have succeeded in observing Anderson localization of coherent matter waves, in the groups of A. Aspect in France and here at LENS. In the French experiment (Billy et al., 2008) a Bose-Einstein condensate is left free to expand in a disordered waveguide produced by combining a weakly focused laser beam with a 1D speckle potential. Differently from the conceptually similar experiments Clément et al. (2005) and Fort et al. (2005), the analysis of the in-situ density profiles shows clear indication of exponentially decreasing tails, which is a signature of Anderson localization. This have been made possible by a combination of several factors: the small atomic density in the tails (necessary to reduce the counteracting effect of interactions), the small speckle autocorrelation length (necessary to have many quantum scattering events during the diffusion) and the high detection sensitivity (allowing the observation of exponential decay of the density). The observed localization is then quantitatively compared with the theory developed in Sanchez-Palencia et al. (2007, 2008).

In the experiment at LENS (Roati et al., 2008), Anderson localization has been observed for a noninteracting ^{39}K BEC in an incommensurate bichromatic lattice, similar to that used in Lye et al. (2007). Here the strategy to exclude the effect of interactions is different: instead of working with dilute samples, interactions are cancelled by tuning a static magnetic field in proximity of a Feshbach resonance to set the scattering length to zero. The noninteracting condensate in the quasiperiodic potential thus realizes the noninteracting tight-binding Aubry-André model of Eq. (8), which exhibits a transition from extended to localized states for increasing disorder. The crossover between extended to localized states is studied in detail by looking at the expansion of the BEC and by studying spatial and momentum distribution of the states, all of which result in agreement with the Aubry-André predictions.

4 Strongly interacting regime

In the previous section we have discussed the physics of disordered weakly-interacting bosonic systems. The theoretical description of this regime is provided by the semiclassical Gross-Pitaevskii Eq. (7), which describes the propagation of nonlinear matter waves. When interactions are strong, however, this mean-field description is not capable to fully explain the behavior of the system. A more appropriate description is provided by a full quantum theory, taking into account quantum correlations between particles. Also in this strongly interacting regime disorder may induce localized quantum phases: these have a different nature from Anderson localization since correlations between particles are important, whereas Anderson localization is essentially a single-particle effect.

Experimentally, a convenient way to enter the strongly interacting regime is provided by the use of optical lattices (which we have already introduced in Sec. 2.2). In a deep optical lattice the system becomes effectively stronger-interacting because of the combined effect of the tighter squeezing of the atom wavefunction in the potential wells (with a consequent increase of the local density) and of the increase in the effective mass due to the finite tunneling times across the potential barriers (which makes the kinetic energy less important with respect to the interaction energy).

For a system defined on a lattice, starting from the full many-body Hamiltonian, one can derive a simplified zero-temperature model, in the approximation that all the particles occupy the fundamental vibrational state of the lattice sites. In this limit the quantum state of an interacting gas of identical bosons in a lattice potential is well described by the second quantization Bose-Hubbard Hamiltonian (Fisher et al., 1989; Jaksch et al., 1998)

$$\hat{H} = -J \sum_{\langle j,j' \rangle} \hat{b}_j^\dagger \hat{b}_{j'} + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) + \sum_j \epsilon_j \hat{n}_j \quad (9)$$

where \hat{b}_j (\hat{b}_j^\dagger) is the annihilation (creation) operator of one particle in the j -th site, $\hat{n}_j = \hat{b}_j^\dagger \hat{b}_j$ is the number operator, and $\langle j,j' \rangle$ indicates the sum on nearest neighbors. Each of the three terms on the right-hand-side of Eq. (9) accounts for a different contribution to the total energy of the system: J is the *hopping energy*, proportional to the probability of quantum tunneling of a boson between neighboring sites, U is the on-site *interaction energy*, arising from atom-atom on-site short-range interactions (repulsive for ^{87}Rb , for which $U > 0$) and giving a nonzero contribution only if more than one particle occupies the same site, and $\epsilon_j \in [-\Delta/2, \Delta/2]$ is a site-dependent energy accounting for inhomogeneous external potentials superimposed on the lattice.

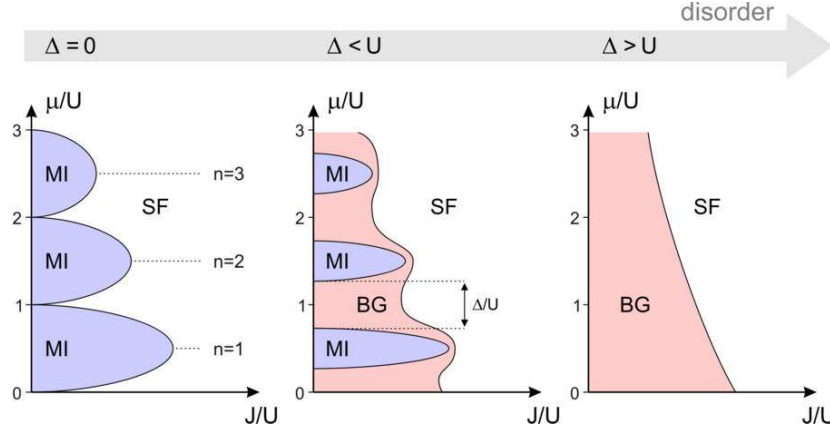


Fig. 14. Qualitative phase diagram for a disordered system of lattice interacting bosons. Three phases can be identified: a superfluid (SF), a Mott insulator (MI) and a Bose glass (BG).

The quantum phase of the system depends on the interplay between these three energy scales: hopping energy J , interaction energy U and disorder Δ . We start considering the ideal case of a translationally invariant system, in which $\Delta = 0$. Assuming integer filling of the sites, when $J > U$ the system is in a superfluid (SF) state, in which the bosons are delocalized across the lattice and the tunneling ensures off-diagonal long-range coherence. Instead, when $U > J$, the system is in a localized Mott insulator (MI) state, where long-range phase coherence is lost and number Fock states are created at the lattice sites. The actual phase diagram of the system depends on the chemical potential (related to the atomic density) and shows the existence of MI lobes with integer number of atoms per site (Fisher et al., 1989). In the left graph of Fig. 14 we show a qualitative sketch of the phase diagram for a 3D system.

The transition from a SF to a MI for ultracold bosons in an optical lattice has been proposed in Jaksch et al. (1998) and reported for the first time in Greiner et al. (2002), where the ratio J/U was varied across the transition point by controlling the height of the lattice. The realization of a MI state does require a 3D optical lattice, since, in order to enter the strongly interacting regime, the atomic wavefunction should be squeezed in tightly confining traps, with a site occupation on the order of unity. However, by using deep optical lattices effectively slicing the atomic sample into decoupled 2D or 1D systems, it is possible to study the SF-MI transition in lower dimensionality, as made in Stöferle et al. (2004) and Spielman et al. (2007).

In the presence of a disordered external potential the additional energy scale Δ enters the description of the system and is responsible for the existence of a new quantum phase. In the presence of weak disorder the MI lobes in the phase diagram should progressively shrink and a new *Bose glass* (BG) phase

should appear (central graph of Fig. 14), eventually washing away the MI region for $\Delta > U$ (right graph of Fig. 14) (Fisher et al., 1989). In a simplified view, a Bose glass is half-way from a Mott insulator to a superfluid: it is an insulating state, with no long-range phase coherence, as the Mott insulator is; nevertheless, it is compressible and has no energy gap in the excitation spectrum, as a superfluid has.

The Bose glass phase has been first identified in Giamarchi and Schulz (1988), where strongly interacting 1D bosonic systems were studied. In the '90s it was widely studied in the context of the superfluid-insulator transition observed in many condensed-matter systems, such as ^4He adsorbed on porous media (Crowell et al., 1995), thin superconducting films (Goldman and Marković, 1998), arrays of Josephson junctions (Van der Zant et al., 1992) and high-temperature superconductors (Jiang et al., 1994; Budhani et al., 1994). The possible realization of a Bose glass in a system of ultracold bosons in a disordered lattice has been first proposed in Damski et al. (2003); Roth and Burnett (2003). More recently, the phase diagram of this system has been derived in other theoretical papers, considering also finite temperature effects (Krutitsky et al., 2006; Buonsante et al., 2007a), detection schemes (Bar-Gill et al., 2006) and the possible realization of a Bose glass with incommensurate bichromatic lattices (Pugatch et al., 2006; Roscilde, 2007). Evidences for Bose glass phases have been also theoretically obtained for different classes of Bose-Hubbard models, where disorder is introduced either in the hopping energy (Buonsante et al., 2007b) or in the on-site interaction energy (Gimperlein et al., 2005).

The Bose glass is just the simplest disordered quantum phase that can be realized in the strongly interacting regime. When atoms of different species, or different internal (spin) states of the same species, are considered, more complicated models can be experimentally realized and new disordered quantum phases can emerge. Atomic Bose/Fermi mixtures, in particular, represent a versatile system in which many different disordered models can be realized (Sanpera et al., 2004; Ahufinger et al., 2005). In the strong interacting limit this system can be described in terms of composite fermionic particles corresponding to one fermion + one bosonic particle/hole in the same site. Sanpera et al. (2004) have shown that the interaction between these composite fermions can be tuned by changing the external potential: thus, a disordered potential can be used to induce an effective random interaction between the particles. This possibility allows the investigation of a variety of disordered-related models, from fermionic Ising spin glasses to models of quantum percolation (Ahufinger et al., 2005).

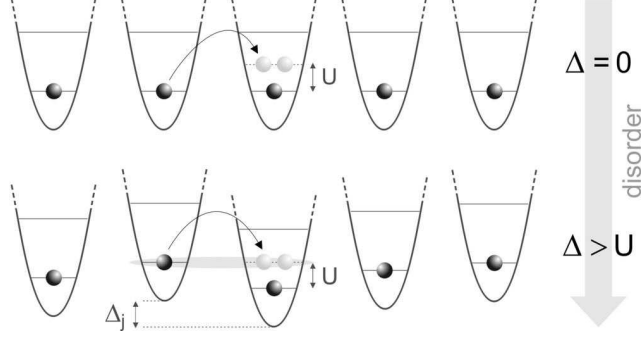


Fig. 15. Excitations in the deep insulating phases. a) In a Mott insulator the tunneling of one boson from a site to a neighboring one has an energy cost $\Delta E = U$. b) In the disordered case the excitation energy is $\Delta E = U \pm \Delta_j$, that becomes a function of the position. In the Bose glass state, in which $|\Delta_j| > U$, an infinite system could be excited at arbitrarily small energies and the energy gap would disappear.

4.1 The quest for Bose glass

Experiments with disordered bosons in the strongly interacting regime started at LENS in 2006. The system under investigation was a collection of 1D atomic systems in a bichromatic optical lattice. A main optical lattice was used to induce the transition from a weakly interacting superfluid to a strongly correlated Mott insulator. A secondary optical lattice was then used to add controlled quasi-disorder to the perfect crystalline structure of the MI phase. With reference to Eq. (9), the non-commensurate periodic potential superimposed on the main lattice introduces inhomogeneities of the energy landscape $\epsilon_j \in [-\Delta/2, \Delta/2]$ on the same length scale as the lattice spacing.

4.1.1 Excitation spectrum and coherence properties

As introduced in the previous section, the excitation spectrum is an important observable that can be measured in order to characterize the quantum state of the system. By exploiting the possibility of time-modulating the lattice potential, as first realized in Stöferle et al. (2004), it is possible to directly measure the excitation spectrum and study how it is modified by the presence of disorder. Naively speaking, in the MI phase one realizes a crystal of atoms pinned at the lattice sites and sitting on the fundamental vibrational level, as schematically shown in the top of Fig. 15. In a MI an energy gap in the excitation spectrum exists, since the elementary excitation - the hopping of a particle from a site to a neighboring one, or, in other words, the creation of a particle-hole pair - has an energy cost U , corresponding to the interaction energy of a pair of mutually repelling atoms sitting on the same site (see Fig. 15).

In Fig. 16a we show the excitation spectrum of a Mott insulator measured in the LENS experiments (Fallani et al., 2007). The plot shows a well resolved resonance at energy U , which is distinctive of the MI state, and a second resonance at energy $2U$. While the physical origin of the excitation peak at U is the tunneling of particles between sites with the same occupancy, the second peak at $2U$ can be ascribed to several processes: it can arise from tunneling at the boundary between MI regions with different site occupancy (that are present due to the inhomogeneity of the confined sample), from higher-order processes and from nonlinear effects due to the strong modulation. A theoretical analysis of the response of the bosonic system to this lattice modulation has been recently reported in Kollath et al. (2006) and Clark and Jaksch (2006).

When increasing disorder the experiment showed a broadening of the resonance peaks, which eventually become undistinguishable when $\Delta \approx U$. As a matter of fact, the presence of disorder introduces random energy differences $\Delta_j \in [-\Delta, \Delta]$ between neighboring sites (see bottom of Fig. 15). As a consequence, the tunneling of a boson through a potential barrier costs $U \pm \Delta_j$, that becomes a function of the position (Guarrera et al., 2007). The excitation energy is not the same for all the bosons, differently from the pure MI case, and the resonances become inhomogeneously broadened, as can be observed in the experimental spectra at weak disorder ($\Delta < U$) shown in Fig. 16b,c (Fallani et al., 2007). This broadening is in agreement with a semi-classical model (Guarrera et al., 2007) and has been recently predicted in theoretical works (Hild et al., 2006; Zakrzewski, 2008), where the authors study the dynamical response of a 1D bosonic gas in a superlattice potential when a periodic amplitude modulation of the lattice is applied.

Eventually, when $\Delta \gtrsim U$, one expects that an infinite system can be excited at arbitrarily small energies and that the energy gap would shrink to zero. When this happens, nearby sites become degenerate and regions of local superfluidity with short-range coherence appear in the system. This novel many-body state in which there is no gap but the system remains globally insulating is a *Bose glass*.

From the experimental point of view, additional information on the nature of the many-body ground state can be acquired by analyzing the density distribution of the atoms released from the lattice after a time-of-flight. Long-range coherence in the sample results in a density distribution with interference peaks at a distance proportional to the lattice wavevector (Pedri et al., 2001). The visibility of these peaks provides a measurement of phase coherence. When increasing the height of the main lattice, a progressive loss of long-range coherence has been reported in Fallani et al. (2007) indicating the transition from a superfluid to an insulating state, also in the presence of disorder. The combination of the excitation spectra measurements and the time-of-flight images indicates that, with increasing disorder, the system realized in Fallani et al.

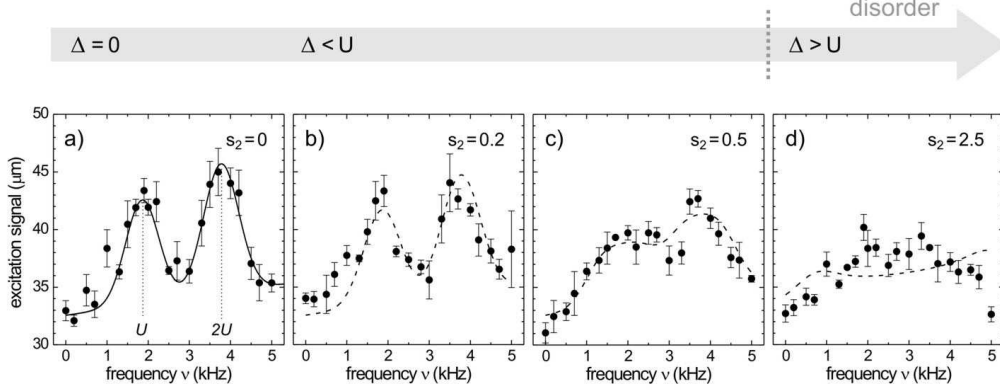


Fig. 16. Excitation spectra of the atomic system in a Mott insulator state for increasing height of the disordering lattice. The resonances are lost and the excitation spectrum becomes flat. Adapted from Fallani et al. (2007).

(2007) goes from a MI to a state with vanishing long range coherence and a flat density of excitations. The concurrence of these two properties cannot be found in either a SF or an ordered MI, and is consistent with the formation of a Bose glass, which is indeed expected to appear for $\Delta \gtrsim U$.

Much work has still to be done for the exhaustive characterization of such novel disordered state. New detection schemes should be implemented, in order to have access to additional observables. This necessity is not only restricted to the study of disordered systems, being a more general issue shared by the experimental investigation of different strongly interacting lattice systems, including e.g. systems with magnetic ordering or mixtures of different species. From the theoretical side, very recent works (Roskilde, 2007; Roux et al., 2008) have extensively studied the problem of 1D interacting bosons in quasiperiodic lattices, working out the phase diagrams (which include the presence of Bose glass and incommensurate “band insulating” / “charge density wave” regions) and studying how the different phases affect experimentally detectable signals.

4.1.2 Noise correlations

In the recent work Guarrera et al. (2008) noise interferometry has been used to study interacting ^{87}Rb bosons in the bichromatic lattice. This detection technique, originally proposed in Altman et al. (2004), is based on the analysis of the spatial density-density correlations of the atomic shot noise after time-of-flight. These correlations are based on the Hanbury Brown & Twiss effect (Hanbury Brown and Twiss, 1956): if two identical particles are released from two lattice sites, the joint probability of detecting them in two separate positions (e.g. imaging them on two separate pixels of a CCD camera) depends on the distance between the detection points. These correlations, arising from

quantum interference between different detection paths, were first observed for bosons in a Mott insulator state (Fölling et al., 2005) and then also for band-insulating fermions (Rom et al., 2006). The sign of the correlations depends on the quantum statistics: while bosons show positive correlations (due to their tendency to bunch, i.e. to arrive together at the detectors), fermions exhibit negative correlations (due to the antibunching, consequence of the Pauli exclusion principle). In the case of a bosonic Mott insulator, one observes positive density-density correlation peaks at a distance proportional to the lattice wavevector k_1 (Fölling et al., 2005), as shown in the first image of the bottom row of Fig. 17 for the recent experiment at LENS.

In Guarrera et al. (2008) noise correlations have been measured, starting from a Mott insulator state, for increasing heights s_2 of the secondary lattice. The absorption images after time-of-flight do not present significative differences, as shown in the top row of Fig. 17, and demonstrate the absence of first order (phase) coherence of the atomic system in the insulating state, even in the presence of the secondary lattice. However, second order (density) correlations turn out to be significantly different with varying s_2 , as illustrated in the noise correlation functions plotted in the bottom row. More precisely, with increasing s_2 , one observes the appearance of additional correlation peaks at a distance proportional to the wavevector k_2 of the secondary lattice and to the beating between the two lattices $k_1 - k_2$. These peaks have to be associated with the redistribution of atoms in the lattice sites as the disordering lattice is strengthened: the MI regions characterized by uniform filling are destroyed and atoms rearrange in the lattice giving rise to a state with non-uniform site occupation, which follows the periodicity of the secondary lattice. The redistribution of atoms is then quantitatively detected by measuring the height of the additional correlation peaks.

Noise correlations thus prove to be a tool to extract important information on the lattice site occupation, which is connected to the second-order correlation function of the many-body state. The appearance of similar correlation peaks was predicted in theoretical works for hard-core bosons (Rey et al., 2006) and soft-core bosons (Roskilde, 2007) in bichromatic lattices. Future works will study the possibility to use noise interferometry to get additional insight on the nature of the disordered insulating states produced in the experiment, in particular in connection with the realization of a Bose glass phase.

4.2 *Experiments with atomic mixtures*

As we have discussed in Sec. 2.3, disorder can be produced by letting the atoms interact with randomly-distributed scatterers of a different atomic species. The configuration proposed in Gavish and Castin (2005) has not yet been realized

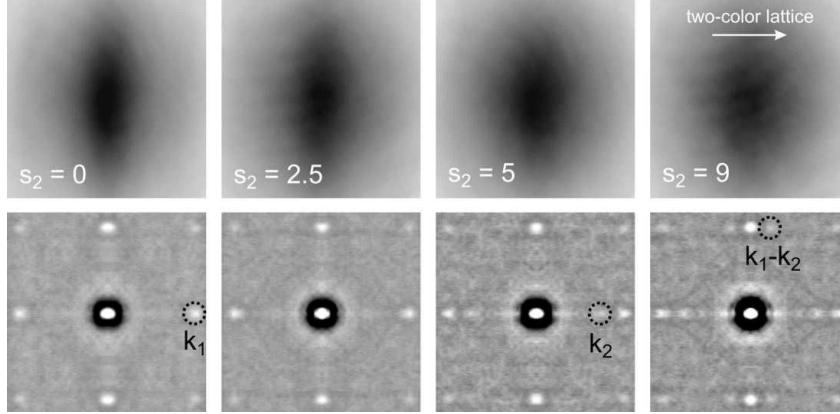


Fig. 17. Top) Time-of-flight absorption images of atoms in a Mott insulator state for increasing height of the secondary lattice s_2 . Bottom) Density-density correlation functions corresponding to the pictures above. The additional correlation peaks for large s_2 arise from the destruction of the Mott domains and the redistribution of the atoms in the lattice. Adapted from Guarrera et al. (2008).

experimentally. However, in 2006 the first experiments with binary mixtures in optical lattices have been realized almost at the same time in two different groups, in Zurich (Günter et al., 2006) and in Hamburg (Ospelkaus et al., 2006). In these experiments ^{87}Rb bosons and ^{40}K fermions were mixed together in a 3D optical lattice. Since the two atoms have very similar resonance wavelengths, the depth of the optical lattice is almost identical for the two species, however, being potassium lighter than rubidium, its mobility is favored. As a result, for a range of lattice heights, rubidium can be localized in a Mott insulator state, while potassium atoms are still able to move across the lattice.

In these experiments the superfluid to Mott insulator transition of ^{87}Rb was investigated as a function of the concentration of ^{40}K impurities, typically in the range 0 to 20%. In particular, the visibility of the interference pattern after time-of-flight was investigated. The observation reported by the two groups was a downshift of the lattice height value at which coherence starts to be lost, when potassium atoms are introduced in the system. Different interpretations for this effect have been given, including finite-temperature effects, disorder-like induced localization, or effects connected with the strong attractive interaction between the two species.

Recently, a closely related system has been investigated at LENS (Catani et al., 2007) by using a binary bosonic mixture of ^{87}Rb and ^{41}K in a 3D optical lattice. Similarly to the experiments described above, loss of coherence in the rubidium sample induced by the presence of potassium has been observed. However, this latter experiment differs from the former ones in two points: the mixture is bosonic/bosonic (instead of bosonic/fermionic) and the interspecies interaction is repulsive (instead of attractive). The observation of similar effects in

systems with different quantum statistics and different interaction sign rules out some of the interpretation given so far, even if a clear explanation of the observation has not yet been found. Future advances of these experiments with mixtures in optical lattices will be pushed by the use of Feshbach resonances for fine tuning of the interspecies interaction.

5 Conclusions

The investigation of Bose-Einstein condensates in disordered potentials is a fastly growing field of research. For decades condensed-matter physicists have theoretically studied the interplay between disorder and interactions in determining the transition from metals to insulators. Now disordered systems can be realized in cold atoms laboratories and, differently from traditional solid state systems, they allow a fine tuning of both disorder and interactions, as well as the advantage of new detection capabilities, thus extending the range of the possible experimental investigations.

The first experiments were realized only a few years ago, with the successful creation of disordered and quasi-disordered potentials and the first studies of the behavior of ensembles of ultracold bosons in different regimes of interactions. Although this field of research is quite young, it already relies on an extensive literature, mostly comprising theoretical works. The physics of disordered atomic systems is indeed extremely rich, both in the weakly interacting regime (where Anderson localization and its disruption by interactions can be studied) and in the strongly interacting regime (where particles are strongly correlated and new quantum localized phases can emerge, as the Bose glass). The few experiments performed until now have just opened a new direction, showing the great potentialities of ultracold atoms for investigating the physics of disorder, but still leaving open questions concerning the observed localization effects. Regarding this point, much more results are likely to come in the near future.

We have tried to give an overview of this newborn field of research, discussing the topics of interest and the experimental efforts made up to now. We would like to conclude by noting that any review is by necessity incomplete and cannot be exhaustive of all the work made in the field. For this reason we apologize with the authors of works which we have unintentionally forgotten to mention.

6 Acknowledgments

This work has been supported by European projects SCALA and EURO-QUAM. We would like to thank D. Clément and M. Modugno for careful reading of the manuscript and all the other members of the Cold Quantum Gases group in Florence. We also acknowledge the authors of the works described in this review who have kindly granted the permission for using their figures.

References

- E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, (1979) Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions, *Phys. Rev. Lett.* 42, 673.
- V. Ahufinger, L. Sanchez-Palencia, A. Kantian, A. Sanpera, and M. Lewenstein, (2005) Disordered ultracold atomic gases in optical lattices: A case study of Fermi-Bose mixtures, *Phys. Rev. A* 72, 063616.
- E. Akkermans, S. Ghosh, and Z. Musslimani, (2008) Numerical study of one-dimensional and interacting Bose-Einstein condensates in a random potential, *J. Phys. B* 41, 045302.
- E. Altman, E. Demler, and M. D. Lukin, (2004) Probing many-body states of ultracold atoms via noise correlations, *Phys. Rev. A* 70, 013603.
- M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, (2005) Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor, *Science* 269, 198.
- P. W. Anderson, (1958) Absence of Diffusion in Certain Random Lattices”, *Phys. Rev.* 109, 1492.
- P. W. Anderson, (1978) Local moments and localized states”, *Rev. Mod. Phys.* 50, 191.
- S. Aubry and G. André, (1980) Analyticity breaking and Anderson localization in incommensurate lattices *Ann. Israel Phys. Soc.* 3, 133.
- N. Bar-Gill, R. Pugatch, E. Rowen, N. Katz, and N. Davidson, (2006) Quantum Phases of Ultra Cold Bosons in Incommensurate 1D Optical Lattices, preprint arXiv:cond-mat/0603513.
- N. Bilas and N. Pavloff, (2006) Anderson localization of elementary excitations in a one-dimensional Bose-Einstein condensate, *Eur. Phys. J. D* 40, 387.
- J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan, D. Clément, L. Sanchez-Palencia, P. Bouyer, and A. Aspect, (2008) Direct observation of Anderson localization of matter-waves in a controlled disorder, preprint arXiv:0804.1621.
- I. Bloch, J. Dalibard, and W. Zwerger, (2007) Many-Body Physics with Ultracold Gases, *Rev. Mod. Phys.* (to be published), preprint arXiv:0704.3011.

- D. Boiron, C. Mennerat-Robilliard, J.-M. Fournier, L. Guidoni, C. Salomon, and G. Grynberg, (1999) Trapping and cooling cesium atoms in a speckle field, *Eur. Phys. J. D* 7, 373.
- R. C. Budhani, W. L. Holstein, and M. Suenaga, (1994) Columnar-defect-induced resistivity minima and Bose glass scaling of linear dissipation in $\text{Ti}_2\text{Ba}_2\text{CaCu}_2\text{O}_8$ epitaxial films, *Phys. Rev. Lett.* 72, 566.
- P. Buonsante, V. Penna, A. Vezzani, and P. B. Blakie, (2007a) Mean-field phase diagram of cold lattice bosons in disordered potentials, *Phys. Rev. A* 76, 011602(R).
- P. Buonsante, F. Massel, V. Penna, and A. Vezzani, (2007b) Mean-field phase diagram for Bose-Hubbard Hamiltonians with random hopping, *Laser Phys.* 17, 538.
- S. Burger, K. Bongs, S. Dettmer, W. Ertmer, K. Sengstock, A. Sanpera, G. V. Shlyapnikov, and M. Lewenstein, (1999) Dark Solitons in Bose-Einstein Condensates, *Phys. Rev. Lett.* 83, 5198.
- F. S. Cataliotti, L. Fallani, F. Ferlaino, C. Fort, P. Maddaloni, and M. Inguscio, (2003) Superfluid current disruption in a chain of weakly coupled Bose-Einstein condensates, *New J. Phys.* 5, 71.
- J. Catani, L. De Sarlo, G. Barontini, F. Minardi, and M. Inguscio, (2008) Degenerate Bose-Bose mixture in a three-dimensional optical lattice, *Phys. Rev. A* 77, 011603(R).
- I. Carusotto, L. Pitaevskii, S. Stringari, G. Modugno, and M. Inguscio, (2005) Sensitive Measurement of Forces at the Micron Scale Using Bloch Oscillations of Ultracold Atoms, *Phys. Rev. Lett.* 95, 093202.
- Y. P. Chen, J. Hitchcock, D. Dries, M. Junker, C. Welford, and R. G. Hulet, (2008) Phase Coherence and Superfluid-Insulator Transition in a Disordered Bose-Einstein Condensate, *Phys. Rev. A* 77, 033632.
- S. R. Clark and D. Jaksch, (2006) Signatures of the superfluid to Mott-insulator transition in the excitation spectrum of ultracold atoms, *New J. Phys.* 8, 160.
- D. Clément, A. F. Varón, M. Hugbart, J. A. Retter, P. Bouyer, L. Sanchez-Palencia, D. M. Gangardt, G. V. Shlyapnikov, and A. Aspect, (2005) Suppression of Transport of an Interacting Elongated Bose-Einstein Condensate in a Random Potential, *Phys. Rev. Lett.* 95, 170409.
- D. Clément, A. F. Varón, J. A. Retter, L. Sanchez-Palencia, A. Aspect, and P. Bouyer, (2006) Experimental study of the transport of coherent interacting matter-waves in a 1D random potential induced by laser speckle, *New J. Phys.* 8, 165.
- D. Clément, P. Bouyer, A. Aspect, and L. Sanchez-Palencia, (2008) Density modulations in an elongated Bose-Einstein condensate released from a disordered potential, *Phys. Rev. A* 77, 033631.
- Ph. W. Courteille, B. Deh, J. Fortágh, A. Günther, S. Kraft, C. Marzok, S. Slama, and C. Zimmermann, (2006) Highly versatile atomic micro traps generated by multifrequency magnetic field modulation, *J. Phys. B: At. Mol. Opt. Phys.* 39, 1055.

- M. Cristiani, O. Morsch, N. Malossi, M. Jona-Lasinio, M. Anderlini, E. Courtade, and E. Arimondo, (2004) Instabilities of a Bose-Einstein condensate in a periodic potential: an experimental investigation, *Opt. Express* 12, 4.
- P. A. Crowell, F. W. Van Keuls, and J. D. Reppy, (1995) “Superfluid-Insulator Transition in ^4He Films Adsorbed in Vycor Glass”, *Phys. Rev. Lett.* **75**, 1106.
- F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, (1999) Theory of Bose-Einstein condensation in trapped gases, *Rev. Mod. Phys.* 71, 463.
- B. Damski, J. Zakrzewski, L. Santos, P. Zoller, and M. Lewenstein, (2003) Atomic Bose and Anderson Glasses in Optical Lattices, *Phys. Rev. Lett.* 91, 080403.
- B. DeMarco, (2007) private communication.
- L. Deng, E. W. Hagley, J. Wen, M. Trippenbach, Y. B. Band, P. S. Julienne, J. E. Simsarian, K. Helmerson, S. L. Rolston, and W. D. Phillips, (1999) Four-wave mixing with matter waves, *Nature* 398, 218.
- R. B. Diener, G. A. Georgakis, J. Zhong, M. Raizen, and Q. Niu, (2001) Transition between extended and localized states in a one-dimensional incommensurate optical lattice, *Phys. Rev. A* 64, 033416.
- S. Drenkelforth, G. Kleine Büning, J. Will, T. Schulte, N. Murray, W. Ertmer, L. Santos, and J. J. Arlt, (2008) Damped Bloch Oscillations of Bose-Einstein Condensates in Disordered Potential Gradients, preprint arXiv:0801.3437.
- J. Estève, C. Aussibal, T. Schumm, C. Figl, D. Mailly, I. Bouchoule, C. I. Westbrook, and A. Aspect, (2004) Role of wire imperfections in micromagnetic traps for atoms, *Phys. Rev. A* 70, 043629.
- G. M. Falco, A. Pelster, and R. Graham, (2007) Collective oscillations in trapped Bose-Einstein-condensed gases in the presence of weak disorder, *Phys. Rev. A* 76, 013624.
- L. Fallani, L. De Sarlo, J. E. Lye, M. Modugno, R. Saers, C. Fort, and M. Inguscio, (2004) Observation of Dynamical Instability for a Bose-Einstein Condensate in a Moving 1D Optical Lattice, *Phys. Rev. Lett.* **93**, 140406.
- L. Fallani, J. E. Lye, V. Guarrera, C. Fort, and M. Inguscio, (2007) Ultracold Atoms in a Disordered Crystal of Light: Towards a Bose Glass, *Phys. Rev. Lett.* 98, 130404.
- M. Fattori, C. D’Errico, G. Roati, M. Zaccanti, M. Jona-Lasinio, M. Modugno, M. Inguscio, and G. Modugno, (2008) Atom Interferometry with a Weakly Interacting Bose-Einstein Condensate, *Phys. Rev. Lett.* 100, 080405.
- R. P. Feynman, (1982) Simulating Physics with Computers, *Int. J. Theor. Phys.* 21, 467.
- M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, (1989) Boson localization and the superfluid-insulator transition, *Phys. Rev. B* 40, 546.
- S. Fölling, F. Gerbier, A. Widera, O. Mandel, T. Gericke, and I. Bloch, (2005) Spatial quantum noise interferometry in expanding ultracold atom clouds, *Nature* 434, 481.
- C. Fort, L. Fallani, V. Guarrera, J. E. Lye, M. Modugno, D. Wiersma, and M. Inguscio, (2005) Effect of Optical Disorder and Single Defects on the

- Expansion of a Bose-Einstein Condensate in a One-Dimensional Waveguide, Phys. Rev. Lett. 95, 170410.
- J. Fortágh, H. Ott, S. Kraft, A. Günther, and C. Zimmermann, (2002) Surface effects in magnetic microtraps, Phys. Rev. A 66, 041604(R).
- U. Gavish and Y. Castin, (2005) Matter-Wave Localization in Disordered Cold Atom Lattices, Phys. Rev. Lett. **95**, 020401.
- T. Giamarchi and H. J. Schulz, (1988) Anderson localization and interactions in one-dimensional metals, Phys. Rev. B 37, 325.
- H. Gimperlein, S. Wessel, J. Schmiedmayer, and L. Santos, (2005) Ultracold Atoms in Optical Lattices with Random On-Site Interactions, Phys. Rev. Lett. 95, 170401.
- A. M. Goldman and N. Marković, (1998) Superconductor-insulator transitions in the two-dimensional limit, Phys. Today 51, 39.
- J. W. Goodman, (2006) Speckle Phenomena in Optics: Theory and Applications, Roberts and Company Publishers.
- M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, (2002) Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms, Nature 415, 39.
- D. R. Grempel, S. Fishman, and R. E. Prange, (1982) Localization in an Incommensurate Potential: An Exactly Solvable Model, Phys. Rev. Lett. 49, 833.
- R. Grimm, M. Weidemüller, and Y. B. Ovchinnikov, (2000) Optical dipole traps for neutral atoms, Adv. At. Mol. Opt. Phys. 42, 95.
- V. Guarrera, L. Fallani, J. E. Lye, C. Fort, and M. Inguscio, (2007) Inhomogeneous broadening of a Mott insulator spectrum, New J. Phys. 9, 107.
- V. Guarrera, N. Fabbri, L. Fallani, C. Fort, K. M. R. van der Stam, and M. Inguscio, (2008) Noise correlation spectroscopy of the broken order of a Mott insulating phase, preprint arXiv:0803.2015.
- K. Günter, T. Stöferle, H. Moritz, M. Köhl, and T. Esslinger, (2006) Bose-Fermi Mixtures in a Three-Dimensional Optical Lattice, Phys. Rev. Lett. 96, 180402.
- M. Gustavsson, E. Haller, M. J. Mark, J. G. Danzl, G. Rojas-Kopeinig, and H.-C. Nägerl, (2008) Control of Interaction-Induced Dephasing of Bloch Oscillations, Phys. Rev. Lett. 100, 080404.
- R. Hanbury Brown and R. Q. Twiss, (1956) Correlation between photons in two coherent beams of light, Nature 177, 27.
- P. G. Harper, (1955) Single band motion of conduction electrons in a uniform magnetic field, Proc. Phys. Soc. A 68, 674.
- M. Hild, F. Schmitt, and R. Roth, (2006) Response of Bose gases in time-dependent optical superlattices, J. Phys. B: At. Mol. Opt. Phys. 39, 4547.
- S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, (1998) "Observation of Feshbach resonances in a Bose-Einstein condensate", Nature 392, 151.
- A. F. Ioffe and A. R. Regel, (1960) Non-crystalline, amorphous, and liquid electronic semiconductors, Prog. Semicond. 237, 4.

- D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, (1998) Cold Bosonic Atoms in Optical Lattices, *Phys. Rev. Lett.* 81, 3108.
- W. Jiang, N.-C. Yeh, D. S. Reed, U. Kriplani, D. A. Beam, M. Konczykowski, T. A. Tombrello, and F. Holtzberg, (1994) Evidence of a Bose-glass transition in superconducting $\text{YBa}_2\text{Cu}_3\text{O}_7$ single crystals with columnar defects, *Phys. Rev. Lett.* 72, 550.
- M. P. A. Jones, C. J. Vale, D. Sahagun, B. V. Hall, C. C. Eberlein, B. E. Sauer, K. Furusawa, D. Richardson, and E. A. Hinds, (2004) Cold atoms probe the magnetic field near a wire, *J. Phys. B: At. Mol. Opt. Phys.* 37, L15.
- C. Kollath, A. Iucci, T. Giamarchi, W. Hofstetter, and U. Schollwöck, (2006) Spectroscopy of Ultracold Atoms by Periodic Lattice Modulations, *Phys. Rev. Lett.* 97, 050402.
- K. V. Krutitsky, A. Pelster, and R. Graham, (2006) Mean-field phase diagram of disordered bosons in a lattice at nonzero temperature, *New J. Phys.* 8, 187.
- J. E. Lye, L. Fallani, M. Modugno, D. Wiersma, C. Fort, and M. Inguscio, (2005) “Bose-Einstein Condensate in a Random Potential”, *Phys. Rev. Lett.* 95, 070401.
- J. E. Lye, L. Fallani, C. Fort, V. Guarrera, M. Modugno, D. S. Wiersma, and M. Inguscio, (2007) Effect of interactions on the localization of a Bose-Einstein condensate in a quasiperiodic lattice, *Phys. Rev. A* 75, 061603.
- A. E. Leanhardt, Y. Shin, A. P. Chikkatur, D. Kielpinski, W. Ketterle, and D. E. Pritchard, (2003) Bose-Einstein Condensates near a Microfabricated Surface, *Phys. Rev. Lett.* 90, 100404.
- P. Lugan, D. Clément, P. Bouyer, A. Aspect, M. Lewenstein, and L. Sanchez-Palencia, (2007a) Ultracold Bose Gases in 1D Disorder: From Lifshits Glass to Bose-Einstein Condensate, *Phys. Rev. Lett.* 98, 170403.
- P. Lugan, D. Clément, P. Bouyer, A. Aspect, and L. Sanchez-Palencia, (2007b) Anderson Localization of Bogolyubov Quasiparticles in Interacting Bose-Einstein Condensates, *Phys. Rev. Lett.* 99, 180402.
- A. Marte, T. Volz, J. Schuster, S. Dürr, G. Rempe, E. G. M. van Kempen, and B. J. Verhaar, (2002) Feshbach Resonances in Rubidium 87: Precision Measurement and Analysis, *Phys. Rev. Lett.* 89, 283202.
- P. Massignan and Y. Castin, (2006) Three-dimensional strong localization of matter waves by scattering from atoms in a lattice with a confinement-induced resonance, *Phys. Rev. A* 74, 013616.
- M. Modugno, (2006) Collective dynamics and expansion of a Bose-Einstein condensate in a random potential, *Phys. Rev. A* 73, 013606.
- O. Morsch and M. Oberthaler, (2006) Dynamics of Bose-Einstein condensates in optical lattices, *Rev. Mod. Phys.* 78, 179.
- O. Morsch, J. H. Müller, M. Cristiani, D. Ciampini, and E. Arimondo, (2001) Bloch Oscillations and Mean-Field Effects of Bose-Einstein Condensates in 1D Optical Lattices, *Phys. Rev. Lett.* 87, 140402.
- N. Mott, (1978) Electrons in glass, *Rev. Mod. Phys.* 50, 203.
- A. Niederberger, T. Schulte, J. Wehr, M. Lewenstein, L. Sanchez-Palencia, and

- K. Sacha, (2008) Disorder-Induced Order in Two-Component Bose-Einstein Condensates, *Phys. Rev. Lett.* 100, 030403.
- S. Ospelkaus, C. Ospelkaus, O. Wille, M. Succo, P. Ernst, K. Sengstock, and K. Bongs, (2006) Localization of Bosonic Atoms by Fermionic Impurities in a Three-Dimensional Optical Lattice, *Phys. Rev. Lett.* 96, 180403.
- P. Pedri, L. Pitaevskii, S. Stringari, C. Fort, S. Burger, F. S. Cataliotti, P. Maddaloni, F. Minardi, and M. Inguscio, (2001) Expansion of a Coherent Array of Bose-Einstein Condensates, *Phys. Rev. Lett.* 87, 220401.
- R. Pugatch, N. Bar-Gill, N. Katz, E. Rowen, and N. Davidson, (2006) Identifying the Bose glass phase, preprint arXiv:cond-mat/0603571.
- M. Raizen, C. Salomon, and Q. Niu, (1997) New Light on Quantum Transport, *Phys. Today* 50, 30 and references therein.
- A. M. Rey, I. I. Satija, and C. W. Clark, (2006) Quantum coherence of hard-core bosons: Extended, glassy, and Mott phases, *Phys. Rev. A* 73, 063610.
- G. Roati, E. de Mirandes, F. Ferlaino, H. Ott, G. Modugno, and M. Inguscio, (2004) Atom Interferometry with Trapped Fermi Gases, *Phys. Rev. Lett.* 92, 230402.
- G. Roati, M. Zaccanti, C. D’Errico, J. Catani, M. Modugno, A. Simoni, M. Inguscio, and G. Modugno, (2007) ^{39}K Bose-Einstein Condensate with Tunable Interactions, *Phys. Rev. Lett.* 99, 010403.
- G. Roati, C. D’Errico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno, and M. Inguscio, (2008) Anderson localization of a non-interacting Bose-Einstein condensate, preprint arXiv:0804.2609.
- T. Rom, Th. Best, D. van Oosten, U. Schneider, S. Fölling, B. Paredes, and I. Bloch, (2006) Free fermion antibunching in a degenerate atomic Fermi gas released from an optical lattice, *Nature* 444, 733.
- T. Roscilde, (2007) Bosons in one-dimensional incommensurate superlattices, preprint arXiv:0712.2741.
- R. Roth and K. Burnett, (2003) Phase diagram of bosonic atoms in two-color superlattices, *Phys. Rev. A* 68, 023604.
- G. Roux, T. Barthel, I. P. McCulloch, C. Kollath, U. Schollwoeck, and T. Giamarchi, (2008) The quasi-periodic Bose-Hubbard model and localization in one-dimensional cold atomic gases, preprint arXiv:0802.3774.
- L. Sanchez-Palencia and L. Santos, (2005) Bose-Einstein condensates in optical quasicrystal lattices, *Phys. Rev. A* 72, 053607.
- L. Sanchez-Palencia, (2006) Smoothing effect and delocalization of interacting Bose-Einstein condensates in random potentials, *Phys. Rev. A* 74, 053625.
- L. Sanchez-Palencia, D. Clément, P. Lugan, P. Bouyer, G. V. Shlyapnikov, and A. Aspect, (2007) Anderson Localization of Expanding Bose-Einstein Condensates in Random Potentials, *Phys. Rev. Lett.* 98, 210401.
- L. Sanchez-Palencia, D. Clément, P. Lugan, P. Bouyer, and A. Aspect, (2008) Disorder-induced trapping versus Anderson localization in Bose-Einstein condensates expanding in disordered potentials, preprint arXiv:0803.1774, *New J. Phys.* (in press).
- A. Sanpera, A. Kantian, L. Sanchez-Palencia, J. Zakrzewski, and M. Lewen-

- stein, (2004) Atomic Fermi-Bose Mixtures in Inhomogeneous and Random Lattices: From Fermi Glass to Quantum Spin Glass and Quantum Percolation, *Phys. Rev. Lett.* 93, 040401.
- T. Schulte, S. Drenkelforth, J. Kruse, W. Ertmer, J. Arlt, K. Sacha, J. Zakrzewski, and M. Lewenstein, (2005) Routes Towards Anderson-Like Localization of Bose-Einstein Condensates in Disordered Optical Lattices, *Phys. Rev. Lett.* 95, 170411.
- T. Schulte, S. Drenkelforth, J. Kruse, R. Tiemeyer, K. Sacha, J. Zakrzewski, M. Lewenstein, W. Ertmer, and J. J. Arlt, (2006) Analysis of localization phenomena in weakly interacting disordered lattice gases, *New J. Phys.* 8, 230.
- T. Schulte, S. Drenkelforth, G. Kleine Büning, W. Ertmer, J. Arlt, M. Lewenstein, and L. Santos, (2008) Dynamics of Bloch Oscillations in Disordered Lattice Potentials, *Phys. Rev. A* 77, 023610.
- T. Schwartz, G. Bartal, S. Fishman, and M. Segev, (2007) Transport and Anderson localization in disordered two-dimensional photonic lattices, *Nature* 446, 52.
- A. Sen De, U. Sen, J. Wehr, and M. Lewenstein, (2007) Classical Spin Models with Broken Continuous Symmetry: Random Field Induced Order and Persistence of Spontaneous Magnetization, preprint arXiv:0712.0829.
- B. Shapiro, (2007) Expansion of a Bose-Einstein Condensate in the Presence of Disorder, *Phys. Rev. Lett.* 99, 060602.
- I. B. Spielman, W. D. Phillips, and J. V. Porto, (2007) Mott-Insulator Transition in a Two-Dimensional Atomic Bose Gas, *Phys. Rev. Lett.* 98, 080404.
- T. Stöferle, H. Moritz, C. Schori, M. Köhl, and T. Esslinger, (2004) Transition from a Strongly Interacting 1D Superfluid to a Mott Insulator, *Phys. Rev. Lett.* 92, 130403.
- H. S. J. van der Zant, F. C. Fritschy, W. J. Elion, L. J. Geerligs, and J. E. Mooij, (1992) Field-induced superconductor-to-insulator transitions in Josephson-junction arrays, *Phys. Rev. Lett.* 69, 2971.
- D.-W. Wang, M. D. Lukin, and E. Demler, (2004) Disordered Bose-Einstein Condensates in Quasi-One-Dimensional Magnetic Microtraps, *Phys. Rev. Lett.* 92, 076802.
- J. Wehr, A. Niederberger, L. Sanchez-Palencia, and M. Lewenstein, (2006) Disorder versus the Mermin-Wagner-Hohenberg effect: From classical spin systems to ultracold atomic gases, *Phys. Rev. B* 74, 224448.
- D. S. Wiersma, M. P. van Albada, B. A. van Tiggelen, and A. Lagendijk, (1995) Experimental Evidence for Recurrent Multiple Scattering Events of Light in Disordered Media, *Phys. Rev. Lett.* 74, 4193.
- D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, (1997) Localization of light in a disordered medium, *Nature* 390, 671.
- B. Wu and Q. Niu, (2001) Landau and dynamical instabilities of the superflow of Bose-Einstein condensates in optical lattices, *Phys. Rev. A* 64, 061603R.
- J. Zakrzewski, (2008) private communication.